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# A COMPUTER PROGRAM FOR AN ANALYSIS OF THE RELATIVE MOTION OF A SPACE STATION AND A FREE FLYING EXPERIMENT MODULE

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Marshall Space Flight Center, Alabama 35812

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## FOREWORD

This report presents the results of work performed by Lockheed's Huntsville Research & Engineering Center while under subcontract to Northrop Nortronics (NSL PO 5-09287) for the Aero-Astrodynamics Laboratory of Marshall Space Flight Center (MSFC), Contract NAS8-20082. This task was conducted in response to the requirement of Appendix E-1, Schedule Order No. E-86.

## ACKNOWLEDGMENT

The author is grateful for the assistance and guidance of the following Lockheed-Huntsville personnel: R. S. Pimm and R. R. Brakensiek. Appreciation is extended to NASA-MSFC personnel for their direction of this study, in particular R. Duncan and A. Schwaniger, S&E-AERO-GA.

## SUMMARY

This report represents the development of a computer program for a preliminary analysis of the relative motion of a "free flying" experiment module in the vicinity of a Space Station under the perturbative effects of drag and earth oblateness. A listing of a computer program developed for determining the relative motion of a module utilizing the Cowell procedure is presented, as well as instructions for its use.

# CONTENTS

## Section 1
## INTRODUCTION

In this decade, large, semi-permanent, manned space stations will be launched. These stations will provide the facilities to study and understand the nature of space as well as the bases for continuously observing the earth and its atmosphere. Experiment modules containing laboratory facilities will operate either attached to the stations or detached or "free flying," depending on requirements of the experiments.

This study was undertaken to develop a computer program to analyze the relative motion of an experiment module and a space station as they travel in orbit. The program considers a specific case in which the module operates in a "free flying" mode near the space station. Program capability is not limited to this specific application, however, as the program can be used in any situation in which the relative motion of two vehicles in nearly the same orbit is desired. For example, "booster-spacecraft" separations can be examined.

In developing the computer program, two approaches for examining relative motion appear: (1) a simplified approach, in which only two-body or Keplerian motion of the module and the station are considered, and (2) a more realistic approach in which are considered deviations in the motion of both vehicles due to the atmosphere and shape of the central body and external forces (such as those of the gravity of the sun and moon, and the sun's radiation pressure). The program as developed in the study contains only the perturbative forces of the earth's shape and its atmosphere. A simple two-body (central force field) relationship could depict the motion of both vehicles, and integrating the force equations would lead to, in both instances, simple elliptical orbits, the planes of which are fixed in inertial space. The real earth is not spherical, however, and it does have an atmosphere, both of which cause perturbations to the Keplerian motion.

A preliminary on-orbit sequence for the station and module which has been proposed is to: (1) detach the module from the station, (2) use a propulsive maneuver to achieve a higher orbit and (3) circularize the module's orbit at some predetermined height above the station. The module is then in its "stationkeeping" position. In this mode, the module is in an orbit nearly identical to that of the station, differing only in height. It will be assumed in the analysis that the module has been placed in the "stationkeeping position." Referring to Fig. 1, this position is shown as location A. Under the combined action of drag and oblateness, the gross motion of the module, relative to the station, is depicted in Fig. 1. Due to a larger semi-major axis, which results in a slower angular rate, the module initially falls behind the station. The larger area-to-mass ratio of the module results in a greater drag force on the module than on the station, resulting in loss of altitude by the module. The above series of events causes the module to move from position A to position B. As the module continues to lose altitude, it reaches position C. At this point, which is the maximum recession distance, the module and the station are at the same altitude. As it continues to lose altitude, the velocity of the module relative to the station increases; it moves to position D, and finally catches up to the station (position E). The module will pass the station unless some maneuver is initiated to return it to its initial position (position A).

In subsequent sections of the document are derived, the coordinates of the module relative to the station, perturbation techniques applicable to the program, and a detailed description of the station-module program.

## Section 2
## COORDINATE SYSTEM FOR EXPRESSING RELATIVE MOTION

A coordinate system which has its origin at the station and moves with the station is used.

In Fig. 2, the positive Y1-axis is along the radius vector to the station from the center of the earth pointing away from earth, the positive Z1-axis is in the direction of the angular momentum vector of the station's orbit and the positive X1-axis is in a direction such as to form a right handed coordinate system.

Referring to Fig. 3, assume unit vectors $\hat{i}, \hat{j}, \hat{k}$, having the directions of the positive X, Y, Z, axes of a three-dimensional rectangular earth-centered system. Assume the position and velocity vectors of both the station and the module are known in this system. Thus given,

$$\vec{R}_s = X_s\hat{i} + Y_s\hat{j} + Z_s\hat{k}$$

$$\vec{V}_s = \dot{X}_s\hat{i} + \dot{Y}_s\hat{j} + \dot{Z}_s\hat{k}$$

$$\vec{R}_m = X_m\hat{i} + Y_m\hat{j} + Z_m\hat{k}$$

$$\vec{V}_m = \dot{X}_m\hat{i} + \dot{Y}_m\hat{j} + \dot{Z}_m\hat{k}$$

where $\vec{R}_s$ and $\vec{V}_s$ are the radius vector and velocity vector of the space station and $\vec{R}_m$, $\vec{V}_m$ are the radius vector and velocity vector of the module.

The angular momentum vector of the station orbit is given by

$$\vec{H}_s = \vec{R}_s \times \vec{V}_s$$

The distance $z$ of the module above or below the plane of the station is given by

$$z = |\vec{R}_m| \cos\varphi$$

where $$\cos\varphi = \vec{H}_s \cdot \vec{R}_m / |\vec{H}_s| \, |\vec{R}_m|$$

thus $$z = \vec{H}_s \cdot \vec{R}_m / |\vec{H}_s|$$

The magnitude of the projection of the module's position vector onto the station's plane is given by

$$R'_m = |\vec{R}_m| \cos(90 - \varphi) = |\vec{R}_m| \sin\varphi$$

Taking $\vec{R}_m \times \vec{H}_s$ results in a vector $\vec{Q}$ which lies in the station's plane and is perpendicular to $R'_m$.

The angle $\theta$ between the $\vec{Q}$ vector and the station's position vector $\vec{R}_s$ is given by

$$\cos\theta = \vec{R}_s \cdot \vec{Q} / |\vec{R}_s| \, |\vec{Q}|$$

Thus the module's x-distance and y-distance can be found by

$$x = R'_m \sin(90 - \theta) = R'_m \cos\theta$$

$$y = R'_m \cos(90 - \theta) - |\vec{R}_s| = R'_m \sin\theta - |\vec{R}_s|$$

As a matter of convention, all positive results will indicate the module to be behind station, higher than or above plane of station; for example, refer to Fig. 3, where the x, y, z distances are shown, x is negative, y is negative, and z is positive. The derivations are not restricted to coplanar orbits.

## Section 3
## METHODS OF COMPUTING RELATIVE MOTION

In Section 2, the coordinates of the module relative to the station were derived referenced to a rectangular coordinate system. A method is now needed to compute these coordinates continuously, including drag and oblateness perturbations.

Two basic classes of methods or perturbation are available: "special perturbations" and "general perturbations." In "special perturbations," accelerations of the disturbed body are integrated by using numerical techniques. Consequently, these methods generate a particular orbit for a particular disturbed body, for particular initial conditions. The methods are ideally suited for calculating orbits having limited duration. Utilizing a step-by-step process, the perturbed orbit is continuously determined. The methods of special perturbations are usually classified according to the formulation of the equations to be integrated. Two examples of these formulations are Cowell's method and Encke's method. The main drawback of special perturbations is that errors accumulate from truncation and roundoff. Truncation error results from the difference between the exact solution of the difference equations which approximate the differential equations themselves, whereas the roundoff error results from the difference between the computed and the exact solutions of the difference equations. In numerical integration these errors are difficult to control.

General perturbations are concerned with analytical methods in which the accelerations are expanded into series and integrated term by term. These methods result in solutions to the equations of motion in the form of symbolic formulas which express the sought-for quantities as explicit functions of either time, constants of the problem or constants of integration. Examples of these formulations are the "variation of coordinates" and "variation of parameters" (usually orbital). The methods of general perturbations are ideally suited for the prediction of orbits extending over many periods. The main disadvantage of such methods is that most contain terms for the effects of the disturbing potential but do not include the effects of drag, or if drag is included, it is a simplified drag model. Atmospheric density is generally expressed only as a simple exponential function of altitude and in some formulations, is applied to the drag equation only at perigee.

A "variation of parameter" (general perturbations) formulation was selected from Ref. 1 and applied to a representative station-module example case. The equations were quite similar to those of Kozai (Ref. 2). Singularities in the equations occur for equatorial orbits, circular orbits, and orbits at the critical inclination. The results from the test case indicated that for perturbed motion, where information from point to point along the perturbed orbits is needed (time intervals of five minutes were used), general perturbation methods are not accurate enough for studying the relative motion between two vehicles in nearly the same orbit. Results from the test case are discussed in Section 5. Equations are presented in Appendix B.

The Cowell (special perturbation) formulation was selected to generate the geocentric rectangular coordinates of the station and the module. From these coordinates, the relative coordinates of the module can be determined as outlined in Section 2.

A numerical integration scheme is used to integrate the total acceleration equations in the Cowell formulation. The method is straightforward, and makes no distinction between the disturbing accelerations and the two-body (central body) accelerations. As a result, many significant figures must be carried in a manner that the disturbing accelerations are not overshadowed by the central body acceleration in the numerical integration procedure. A small integration step size (30 seconds for this analysis) should be used to minimize the truncation error. However, with a small integration step size and a large number of steps, the influence of round-off error will be prominent. Thus, this procedure is restricted to calculation of orbits having a duration of only a few days.

The numerical integration of the equations of the Cowell formulation is performed by a fourth order Runge-Kutta numerical procedure. The equations of the Cowell formulation are given in Appendix A.

# Section 4
# THE STATION-MODULE PROGRAM

## 4.1 DISCUSSION

A program incorporating the Cowell formulation to compute the relative motion of the station and module was developed in double precision for the Univac 1108 (Exec 8) computer system. The two-body relative motion, as well as the perturbed relative motion, is determined in the program. The effect of the gravitational zonal harmonics through the fourth ($J_4$) are considered. The density values for the drag perturbations are computed by the MSFC Modified Jacchia Model Atmosphere (1967) which is recommended in Ref. 3.

## 4.2 INPUT

Initial input to the Station-Module (STA-MOD) program needed to execute the program successfully are the semi-major axis, eccentricity, inclination, ascending node, argument of perigee and true anomaly of the station and the module. This information is input on the first two input cards, respectively. The elements are then transformed to position and velocity coordinates in a geocentric rectangular coordinate system for use in the Cowell scheme. The ballistic coefficients ($C_DA/m$) of the station and module respectively, needed for use in the drag calculations, are input on the third card.

Control of the integration step size in the Cowell scheme, cutoff time, and print time is inserted on input Card 4.

The Modified Julian Date at which the initial orbital elements were determined is input on Card 5. This date is necessary for use in the Jacchia density model to determine the semi-annual variation of density.

Atmospheric density is affected to a great extent by variable solar activity and geomagnetic activity. Consequently it must be corrected to account for these phenomena. The index of solar activity, the 10.7 cm decimetric flux, is input as a function of the year to be utilized in the density calculation. The total number of solar flux values and corresponding year values are input on Card 6, whereas their values are input on Card 7. The geomagnetic activity index, Ap, is an average value determined within the program based on the value of the solar flux.

Plots of the perturbed orbital elements of the station and module, the perturbed coordinates of the module relative to the station, the two-body relative coordinates of the module, and the deviation of module coordinates from two-body behavior are available.

Table 1 gives the input cards necessary to execute the program; Table 2 illustrates a 1108 run request with instructions for plots.

A complete program listing is given in Appendix C.

## 4.3 OUTPUT

Table 3 gives an example of the output from the STA-MOD program. In the first block of data the orbital elements of the station PNUIS, true anomaly, AIPS, semi-major axis, EIPS, eccentricity, FINCPS, inclination, CAPWS, ascending node, SMAWS, argument of perigee, MEANPS, mean anomaly are given. The time in minutes, TTIMEM, and time in days, TTIMED, are also shown.

In the second block of data are given the corresponding elements for the module: PNUIM, AIPM, EIPM, FINCPM, CAPWM, SMAWM, and MEANPM. Time for the module and station is the same.

In the third and final block for a given time is the relative distance, in kilometers, of the module from the station in the x-direction, DELTAX; the y-direction, DELTAY; and the Z-direction, DELTAZ. The deviation in the relative motion of the module from two-body relative motion is shown next. The deviation is given with respect to the three coordinate distances. They are DEVX, DEVY, and DEVZ. A new block of data begins after this block.

Computer plots of the orbital elements and relative motion plots of the modules are obtained as an output. Examples of the plots are given in Section 5.

The output shown in Section 5 (Table 3) was generated by the first two input data cards shown in the program listing.

## Section 5
## RESULTS AND CONCLUSIONS

To illustrate the functional ability of the program, results of representative station module cases are presented.

For the first case, the effects of drag were not considered and a non-circular orbit was chosen. An orbit of this type was selected so that results of using Koelle's general perturbation equations and the Cowell special perturbation technique could be compared. There is no provision in Koelle's equations for drag effects and there is a breakdown in the computations for circular orbits.

The initial orbital elements are:

| | Station | Module |
|---|---|---|
| a | 7642.45 | 7642.655 |
| e | 0.1 | 0.1 |
| i | 55 deg | 55 deg |
| $\Omega$ | 0 deg | 0 deg |
| $\omega$ | 0 deg | 0 deg |
| $\theta$ | 0 deg | 0 deg |

The module is initially .185 km above the station. The stations initial perigee altitude is 500 km (270 n.mi.).

Figures 4 through Fig. 24 depict the results for the above case in which the Cowell special perturbation formulation was used. The time period used (900 minutes) was completely arbitrary. Figures 4, 5, and 6 show the module's x, y and z relative distances versus time, respectively. Figure 7 shows the y-relative distance versus the x-relative distance. This is the view of the module in the plane of the station as viewed from the station. In Figs. 4, 5 and 6 the short period variations are quite evident.

Figures 8, 9 and 10 depict the variation from two-body behavior of the module's relative coordinates. It can be seen that, for this orbit and over the time period given, this variation grows to one on the order of 100 meters for the x-relative coordinate and y-relative coordinate, but of a magnitude of 10 meters for the z coordinates. Since for two-body motion, no motion exists out of plane, Figs. 10 and 6 are identical. Figures 11 through 24 show the perturbed orbital elements of the station and the module.

Figures 25 through 45 represent results for the same case as above using Koelle's general perturbation equations. Figures 25, 26 and 27 show the modules x, y, and z relative distances versus time. Figure 7 gives a view of the module's motion in the plane of the station. When these plots are compared with the corresponding plots generated by using Cowell's formulation, good agreement is found between the y and z relative distances. In Fig. 25 the x-relative coordinate (Koelle's equations) appears to have an additional periodic variation superposed on the "known" short period variation. Many procedures were instituted in an effort to remove this additional wiggle, but to no avail. Figure 28 and Fig. 7, depicting the motion in the station plane, do not agree, because Koelle's x-distance does not match Cowell's x-distance values.

The deviations from two-body behavior (Figs. 29, 30 and 31) agree fairly well with Cowell corresponding plots (Figs. 8, 9, 10) only with the deviation from two-body behavior in the z-relative coordinates. The Koelle's plots (Fig. 29 and 30) do not agree in form or magnitude. Figures 32 through 45 depict the perturbed orbital elements of the station and module. It should be remembered that only the perturbative effects of oblateness were considered in the orbit discussed above.

As an additional example in which drag effects are included, the initial conditions of the station and module (Table 4) were considered. The station is 500 km (270 n.mi.) above the earth and the module is .152 km (500 ft) above the station. The ballistic parameter of the module ($C_D A/M$) is 2.5 times that of the station. The drag coefficient has been taken to be 2.2. Since the date of the initial conditions is a future one, the solar activity index, FTENB, will be a predicted value based on mean of past values. Results to be presented are from Cowell formulation.

Table 4

EXAMPLE CASE 2

| Parameter | Station | Module |
|---|---|---|
| Semi-major Axis, a | 6878.556 km | 6878.7084 km |
| Eccentricity, e | 0.0 | 0.0 |
| Inclination, i | 30.0 deg | 30.0 deg |
| Ascending Node, $\Omega$ | 0.0 deg | 0.0 deg |
| Argument of Perigee, $\omega$ | 0.0 deg | 0.0 deg |
| True Anomaly, v | 0.0 deg | 0.0 deg |
| Ballistic Parameter, $C_D A/m$ | 0.0082 $m^2/kg$ | 0.0205 $m^2/kg$ |

DATE: MAY 1, 1980

The example is a "loop case" where the module falls behind the station and subsequently catches up. Figure 46 depicts behavior under the example conditions, and shows the module leading the station after approximately 3750 minutes. Figure 47 show the y-relative coordinate. The magnitude of the fluctuation about a mean value appears to increase as the module falls below the station. The z-relative coordinate, Fig. 48, tend to fluctuate about the station's plane, returning to this plane in the same terms as it takes the module to complete its loop. Figure 49 illustrates the loop.

Note that this example was taken, in total, from Ref. 4. The Ref. 4 analysis indicated that the maximum recession distance would be (27.8 km) (15 n.mi.) and that the time in the "far-out loop" would be 4.63 days. The 1959 ARDC Density Module, with corrections for solar activity, was used to compute density values. The MSFC Modified Jacchia Model Atmosphere (1967) was used in this analysis.

Figure 50 shows the y-relative distance vs. x-relative distance results utilizing the Earth Orbital Decay program (Ref. 5). This program utilizes a first-order variation of parameters technique, in which, the short period variations have been averaged out. The result is that which would be obtained if a mean line were drawn through the results in Fig. 49. The time for the "far-out loop" from this procedure was 2.75 days. The Jacchia model was used in this procedure also.

The results of the program indicate that the program can be used to investigate the behavior of an experiment module or any other vehicle relative to another moving vehicle. The disparity between the general perturbation scheme and the special perturbation scheme should be resolved. In addition, the addition of some type of propulsive capability would be extremely helpful for program flexibility.

Accuracies or inaccuracies which could be incurred when utilizing the program were not determined.

## REFERENCES

1. Koelle, H. H. (Editor) Handbook of Astronautical Engineering, McGraw-Hill, New York, 1961

2. Kozai, Yoshihide, "The Motion of a Close Earth Satellite," Astronomical Journal, Nov. 1959.

3. Weidner, D. K. (Editor), Space Environment Criteria Guidelines for Use in Space Vehicle Development (1969 Revision), NASA-TMX-53957, 17 October 1969.

4. Martin Marietta-Denver Division, "Advanced Astronomy Missions Concepts ATM Follow-On Study, Vol. II - Scientific Instruments, Configurations, Mission Operations," ED-2002-795, April 1969.

5. Kilgo, H. F., "MSFC/LMSC Satellite Orbit Decay and Orbital Lifetime Program," LMSC-HREC A710725, Lockheed Missiles & Space Co., Huntsville, Ala., June 1965.

## BIBLIOGRAPHY

Escobal, P. R., Methods of Orbit Determination, Wiley, New York, 1965.

Perrine, B. S., Jr., "A Method of Soft Tether Stationkeeping," NASA TM X-53643, George C. Marshall Space Flight Center, Huntsville, Ala., 31 July 1967.

Roy, A. E., Foundations of Astrodynamics, MacMillan, New York, 1965.

Private Discussions with L. D. Mullins, S&E-AERO-MM, National Aeronautics and Space Administration, Marshall Space Flight Center, Huntsville, Ala.

# TABLES

Table 1

STA-MOD PROGRAM INPUT

| Input Card No. | Program Symbol | Definition |
|---|---|---|
| 1 | AIPS, EIPS, FINCPS, CAPWS, SMAWS, PNUIS | Semi-major axis (km), eccentricity, inclination (deg), ascending node (deg), argument of perigee (deg), and true anomaly of station (6E12. 8) |
| 2 | AIPM, EIPM, FINCPM, CAPWM, SMAWM, PNUIM | Semi-major axis (km), eccentricity, inclination (deg), ascending node (deg), argument of perigee (deg), and true anomaly (deg) of module (6E12. 8) |
| 3 | CDAS, CDAM | Ballistic coefficient of station and module ($meter^2$/kg) (2E12. 8) |
| 4 | DT, TCUT, NP | Integration step size (sec) for Cowell method, cutoff time (hr), number of DT's (integer) per print interval (print-out will occur every DT x NP seconds) (2E12. 8, I3) |
| 5 | XJD | Modified Julian date at which initial orbital elements for the station and module were given (E12. 8) |
| 6 | K | An integer which specifies the total number of values to be read on card no. 7 (I3) |
| 7 | FTENB | Table of 81-day mean values of the 10.7 cm. solar flux ($10^{-22}$ watts/$m^2$/cyl/sec). For future flights, predicted values are input. The values are loaded in the order FTENB, decimal year, FTENB, decimal year, etc. up to 100 values and the corresponding year may be loaded |

Table 2

UNIVAC 1108 INSTRUCTIONS FOR PLOTS

**1108 RUN REQUEST & INSTRUCTIONS**

| NAME (LAST & INITIAL) | OP CODE | JOB# | PROD # |
|---|---|---|---|
| BUTLER, J. | 12 | 490540 | |

| BIN# | BLDG# | RM# | RUN-ID | RUN |
|---|---|---|---|---|
| 202 | LMSC | | BUTLER | 1 OF 1 |

| EST. CPU RUN TIME | CORE SIZE | PUNCH | |
|---|---|---|---|
| ___ HRS. 3 MINS. | 32K | | ☒ COMPILE<br>☒ EXECUTE<br>☐ SORT |

| ☐<br>☒ EXEC VIII | LANGUAGE | MAX. PAGES | ☐ SPECIAL FORMS |
|---|---|---|---|
| | FOR | 300 | TYPE COPIES |

DOES THIS JOB HAVE A RESTART PROCEDURE? ☐ YES ☐ NO

| INPUT TAPES | | | OUTPUT TAPES ONLY | | | | |
|---|---|---|---|---|---|---|---|
| REEL NO. | FILE NAME | UNIT | REEL NO. | FILE NAME | LOGIC | UNIT | SAVE |
| | | | | | | | |
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PROGRAMMER COMMENTS:

STATION – MODULE

☐ OVER

| MICRO FILM | | COPIES | | COPY FLO | | OPER. INIT. |
|---|---|---|---|---|---|---|
| #FILES | #FRAMES | P | F | P | F | SEQ.# |
| 1 | 21 | 1 | | ✓ | | |

OPERATOR COMMENTS: ☐ SEE TECH. ☐ SEE OPER.

MSFC - Form 3019 (Rev August 1969) ☐ OVER

# FIGURES

Figures (Continued)

Figures (Continued)

Table 3

PROGRAM OUTPUT

```
*****DESCRIPTION OF DATA OUTPUT*****

STATION TRUE ANOMALY (DEGS)
STATION SEMI-MAJOR AXIS (KMS)          STATION ECCENTRICITY          STATION INCLINATION (DEGS)   STATION LONG OF ASCENDING NODE (DEGS)
STATION ARGUMENT OF PERIGEE (DEGS)     STATION MEAN ANOMALY(DEGS)
STATION TIME (MINS)                    STATION TIME (DAYS)

MODULE TRUE ANOMALY (DEGS)
MODULE SEMI-MAJOR AXIS (KMS)           MODULE ECCENTRICITY           MODULE INCLINATION (DEGS)    MODULE LONG OF ASCENDING NODE (DEGS)
MODULE ARGUMENT OF PERIGEE (DEGS)      MODULE MEAN ANOMALY (DEGS)

DELTAX OF MODULE TO STATION (KMS)      DELTAY OF MOD TO STAT (KM)    DELTAZ OF MOD TO STAT (KM)
DEVIATION OF TWO-BODY-X                DEVIATION OF TWO-BODY-Y       DEVIATION OF TWO-BODY-Z

PNUIS =   .14925709-05
AIPS  =   .76424500+04     EIPS   =   .10000000+00     FINCPS=   .55000008+02     CAPWS =   .00000000
SHAWS =  -.14925711-05     MEANPS=    .00000000
TTIMEM=   .00000000        TTIMED=    .00000000

PNUIM =   .95657695-06
AIPM  =   .76426558+04     EIPM   =   .10000000+00     FINCPM=   .55000008+02     CAPMS =   .00000000
SMAWM =  -.95657708-06     MEANPM=    .00000000

DELTAX=  -.35562536-14     DELTAY=    .18519930+00     DELTAZ=   .00000000
DEVX  =  -.35562536-14     DEVY   =   .18519930+00     DEVZ  =   .00000000

PNUIS =   .19656562+02
AIPS  =   .76404970+04     EIPS   =   .99716031-01     FINCPS=   .54996030+02     CAPWS =  -.11388326-02
SHAWS =   .22450217+00     MEANPS=    .16249196+02
TTIMEM=   .50000000+01     TTIMED=    .34722222-02

PNUIM =   .19655783+02
AIPM  =   .76407028+04     EIPM   =   .99716045-01     FINCPM=   .54996030+02     CAPMS =  -.11386425-02
SMAWM =   .22448332+00     MEANPM=    .16248539+02

DELTAX=   .96226903-01     DELTAY=    .18326814+00     DELTAZ=   .24583714-05
DEVX  =   .96215211-01     DEVY   =   .18321690+00     DEVZ  =  -.31797803-14

PNUIS =   .39151468+02
AIPS  =   .76359436+04     EIPS   =   .99152331-01     FINCPS=   .54986340+02     CAPWS =  -.81242531-02
SHAWS =   .20123813+00     MEANPS=    .32527464+02
TTIMEM=   .10000000+02     TTIMED=    .69444444-02

PNUIM =   .39149922+02
AIPM  =   .76361497+04     EIPM   =   .99152389-01     FINCPM=   .54986342+02     CAPMS =  -.81229381-02
SMAWM =   .20123649+00     MEANPM=    .32526147+02

DELTAX=   .18952974+00     DELTAY=    .17810840+00     DELTAZ=   .21240469-04
DEVX  =   .18949186+00     DEVY   =   .17794208+00     DEVZ  =  -.63595606-14

PNUIS =   .58135172+02
AIPS  =   .76314196+04     EIPS   =   .98816209-01     FINCPS=   .54975792+02     CAPWS =  -.23009800-01
SHAWS =  -.43076000-01     MEANPS=    .48834589+02
TTIMEM=   .15000000+02     TTIMED=    .10416667-01

PNUIM =   .58132922+02
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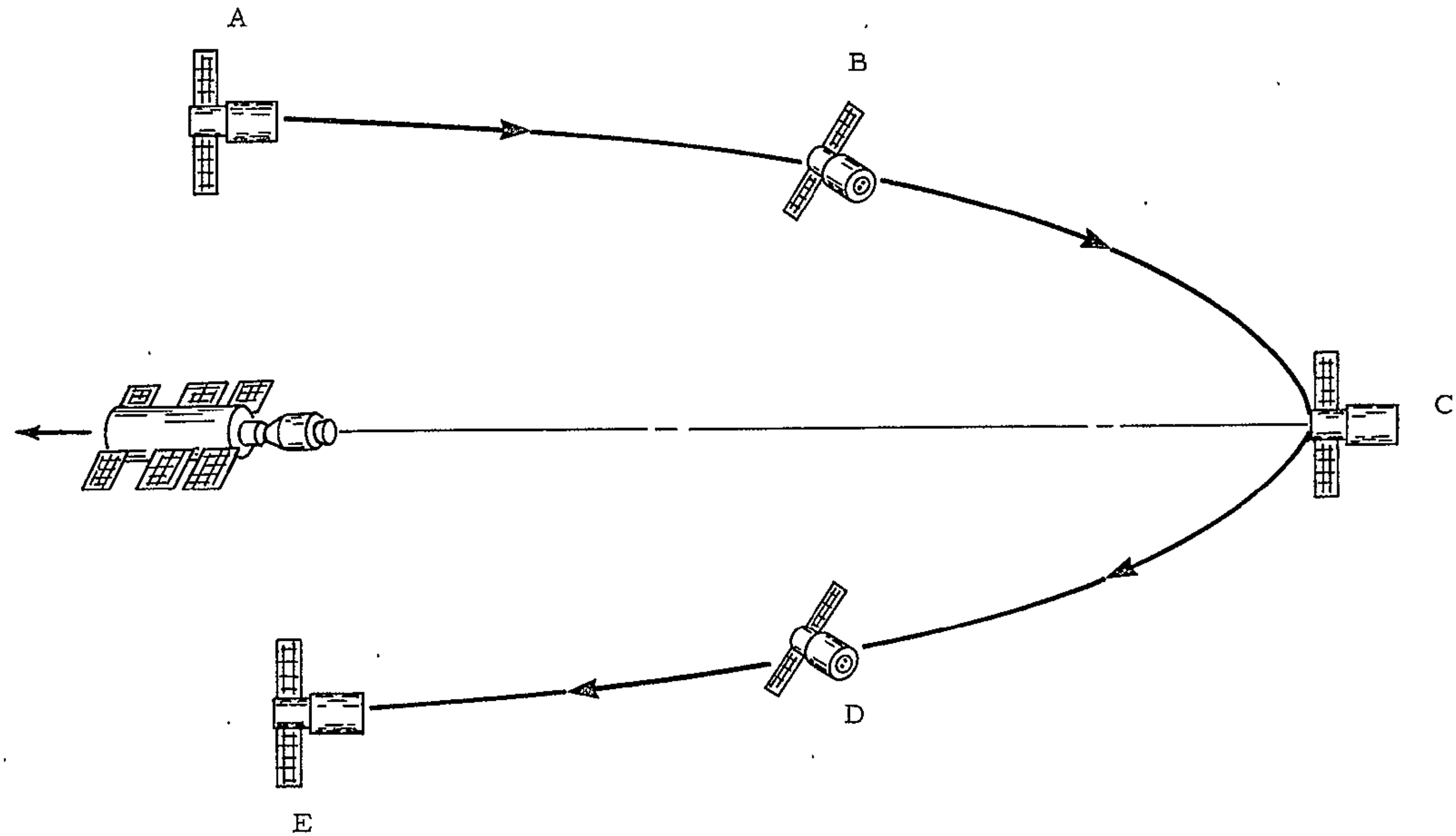


Fig. 1 – Relative Motion of Module with Respect to Space Station

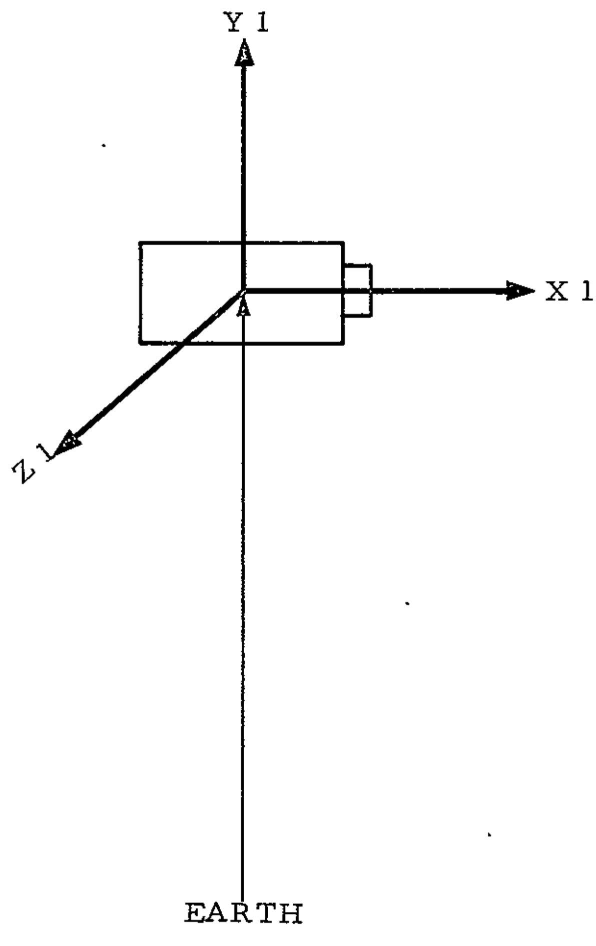


Fig. 2 — Relative Coordinate System

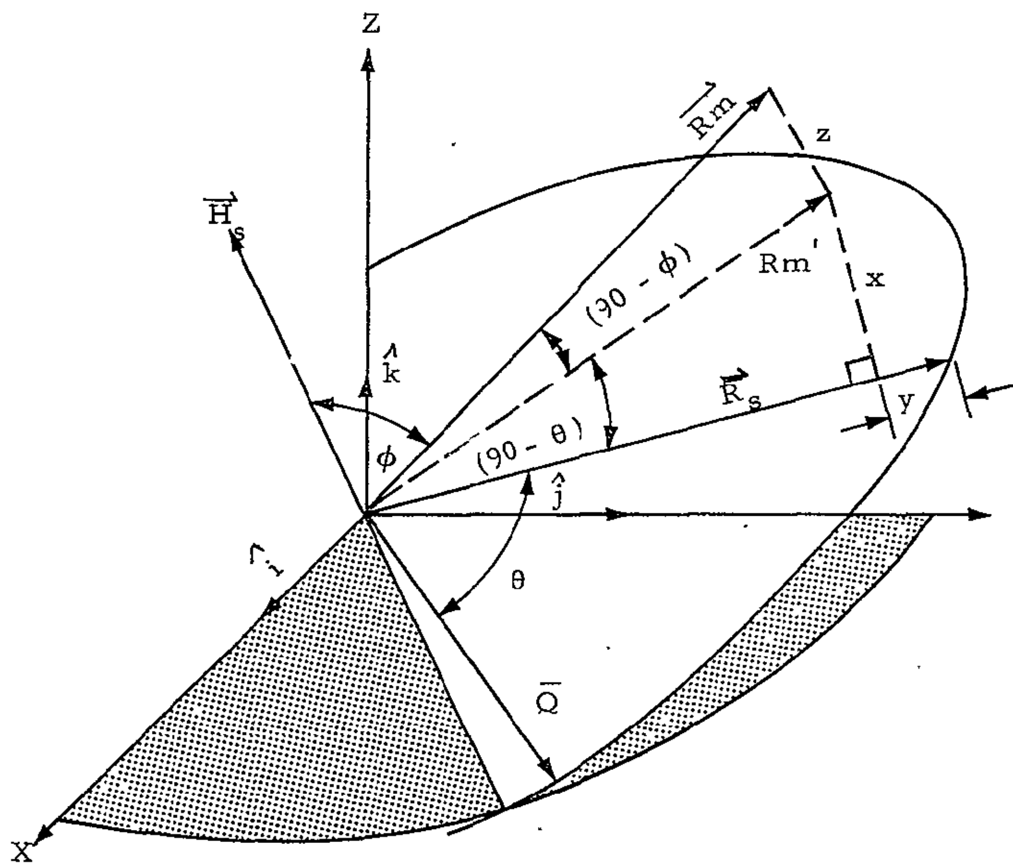


Fig. 3 — Rectangular Earth-Centered System

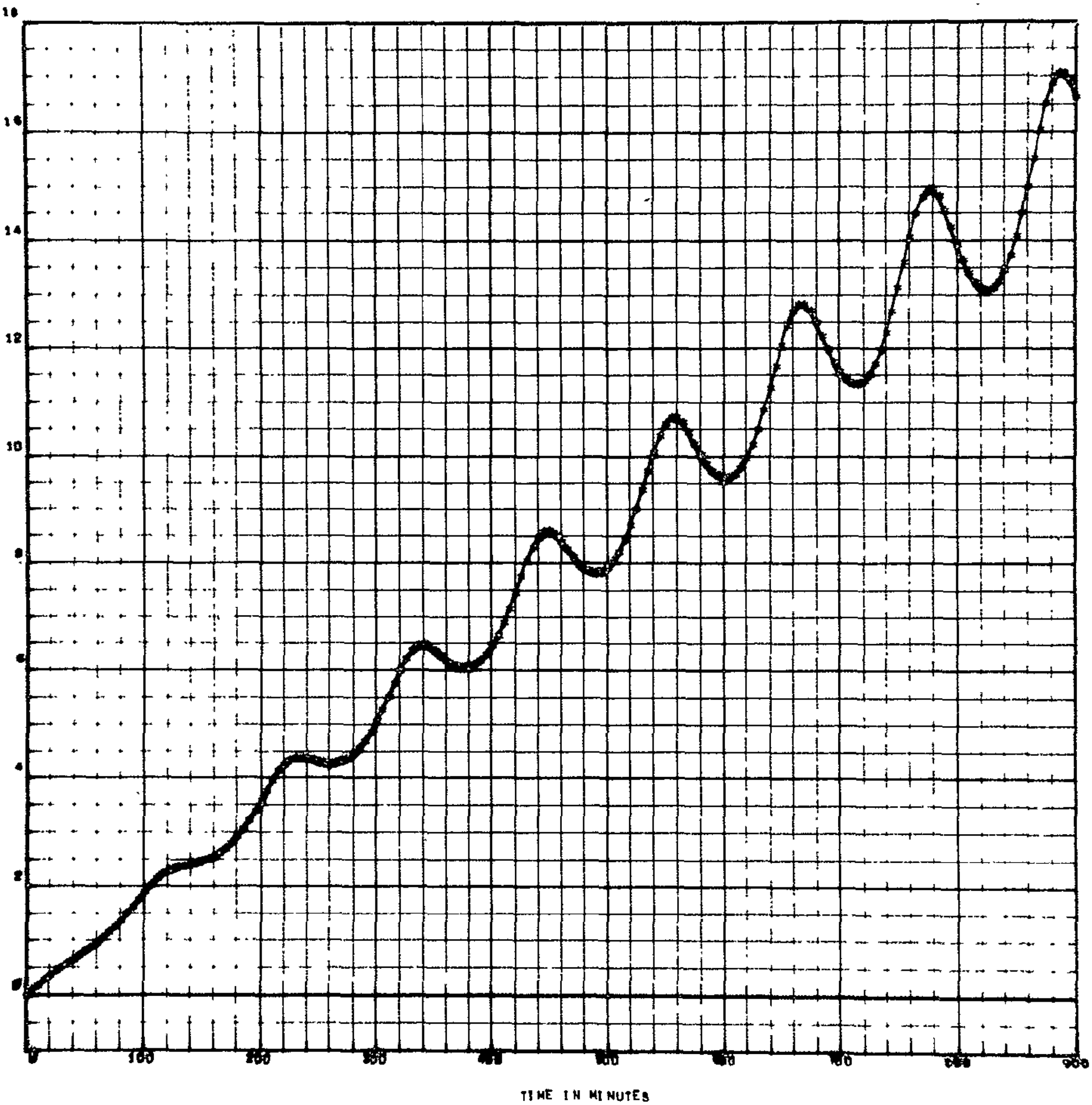


Fig. 4 – Modules X-Relative Position (km) vs. Time (min)

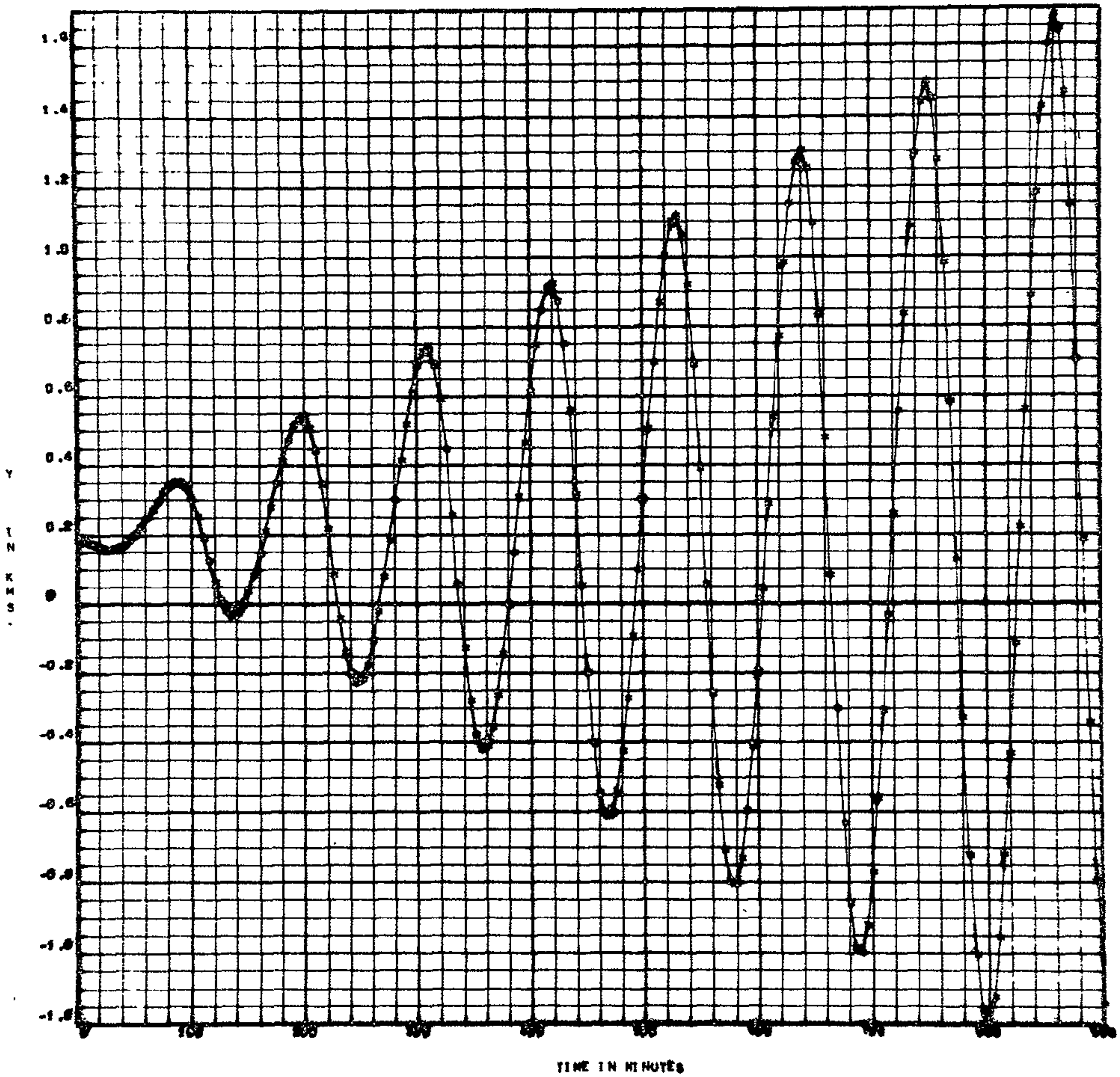


Fig. 5 – Modules Y-Relative Position (km) vs. Time (min)

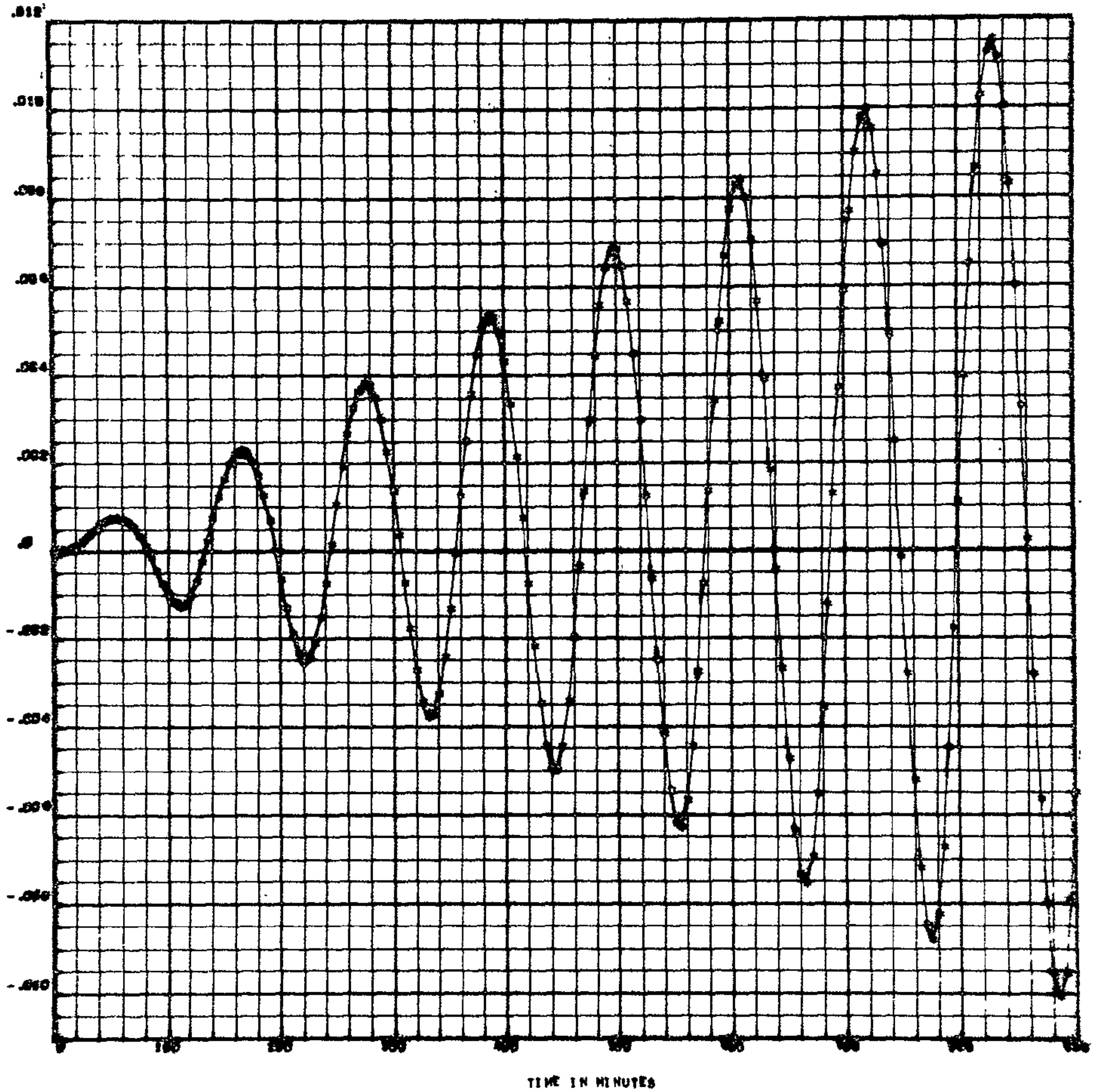


Fig. 6 — Modules Z-Relative Position (km) vs. Time (min)

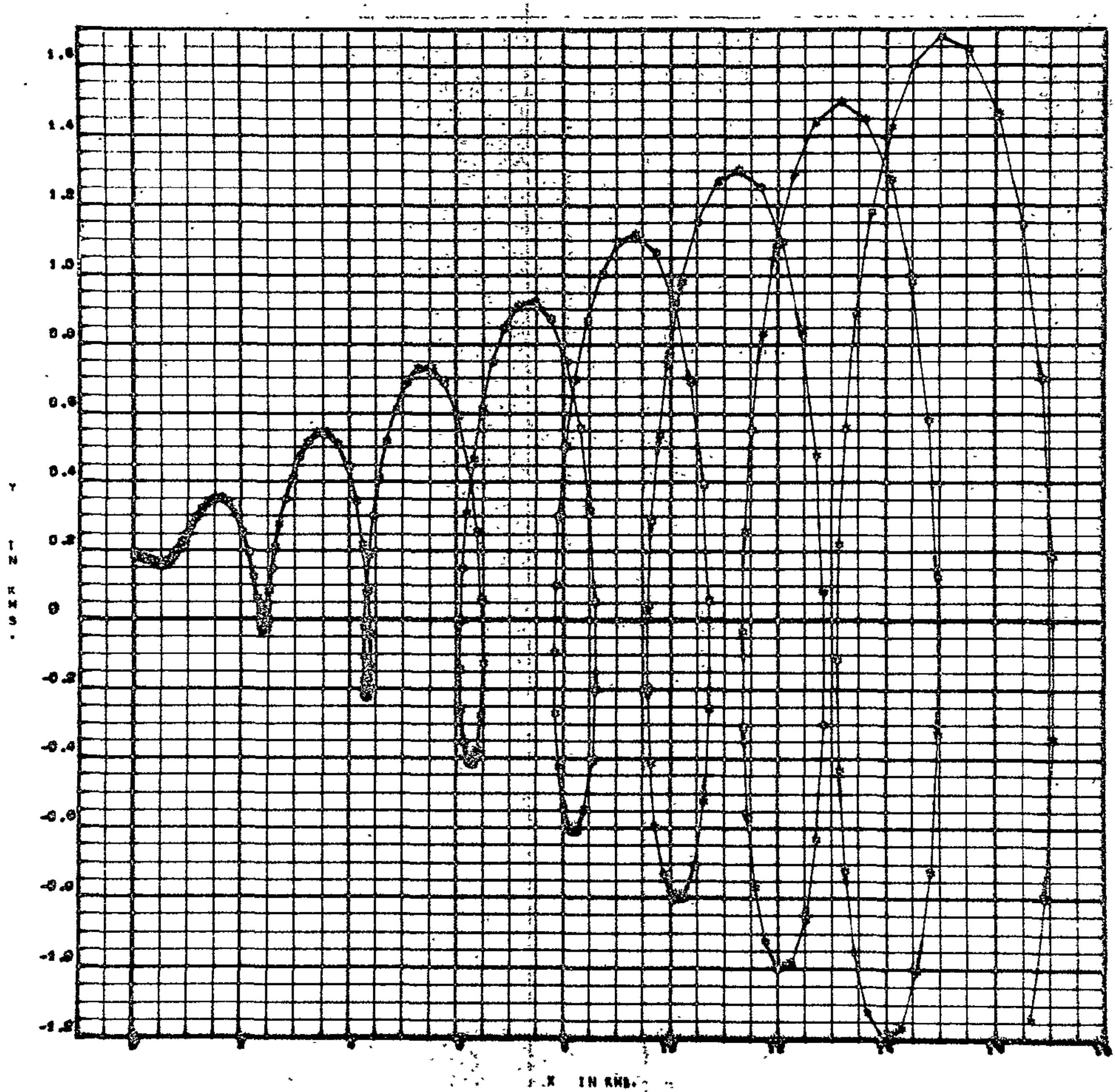


Fig. 7 – Modules Y-Relative Position (km) vs. Modules X-Relative Position (km) – Motion in Station's Plane

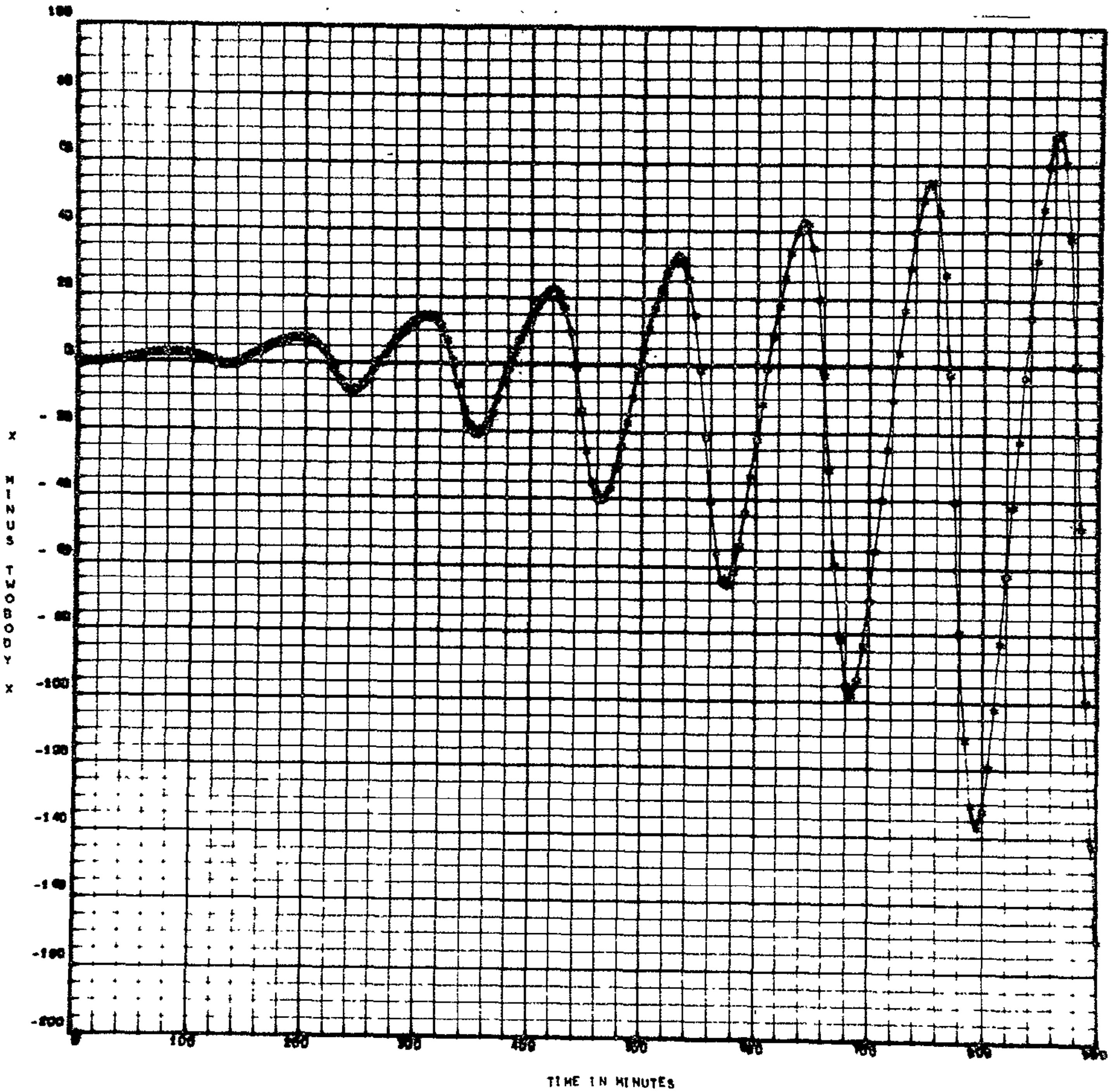


Fig. 8 – Deviation in Modules X-Relative Position from Two-Body Relative X-Position (meters) vs. Time (min)

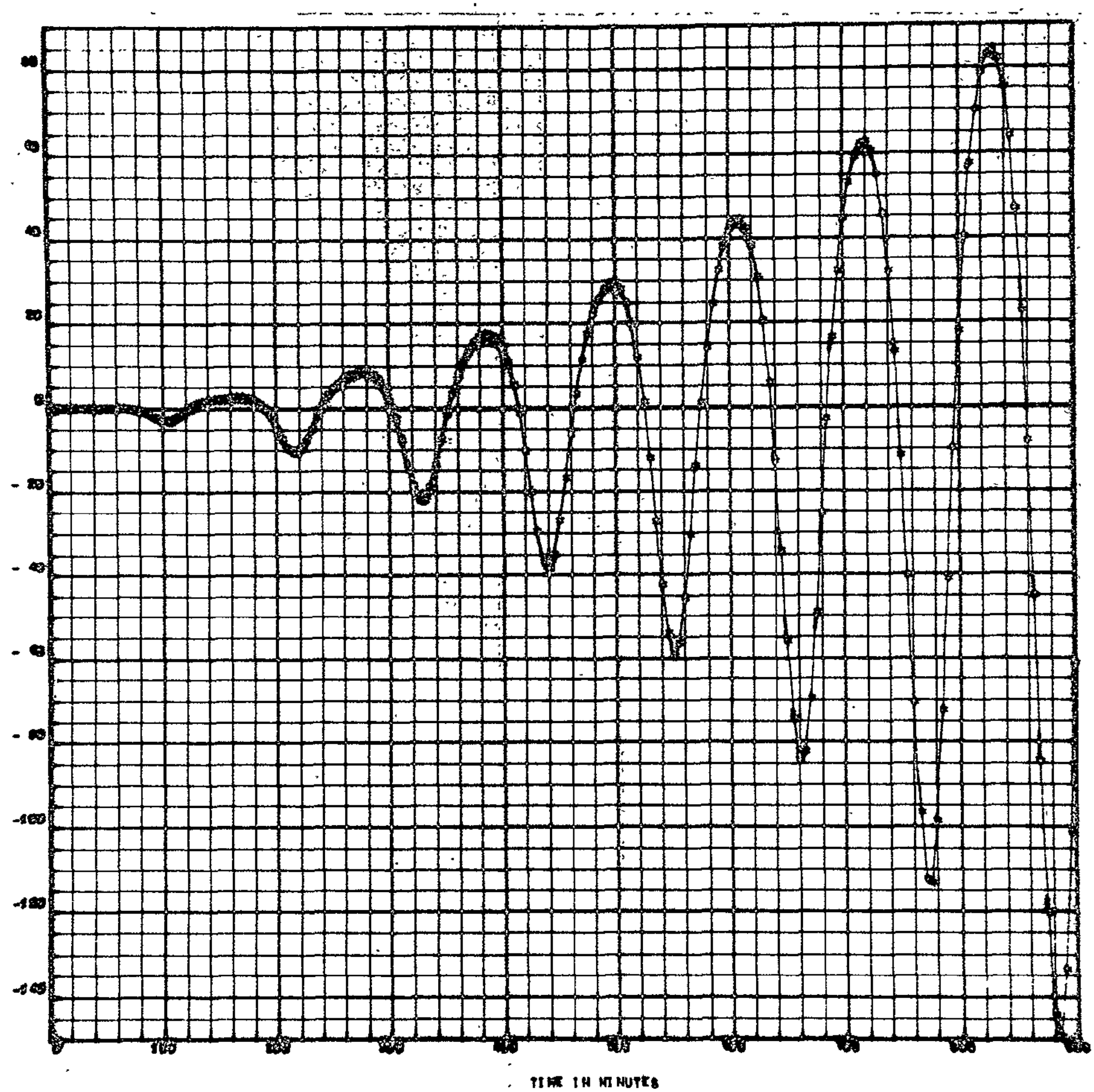


Fig. 9 — Deviation in Modules Y-Relative Position from Two-Body Relative Y Position (meters) vs. Time (min)

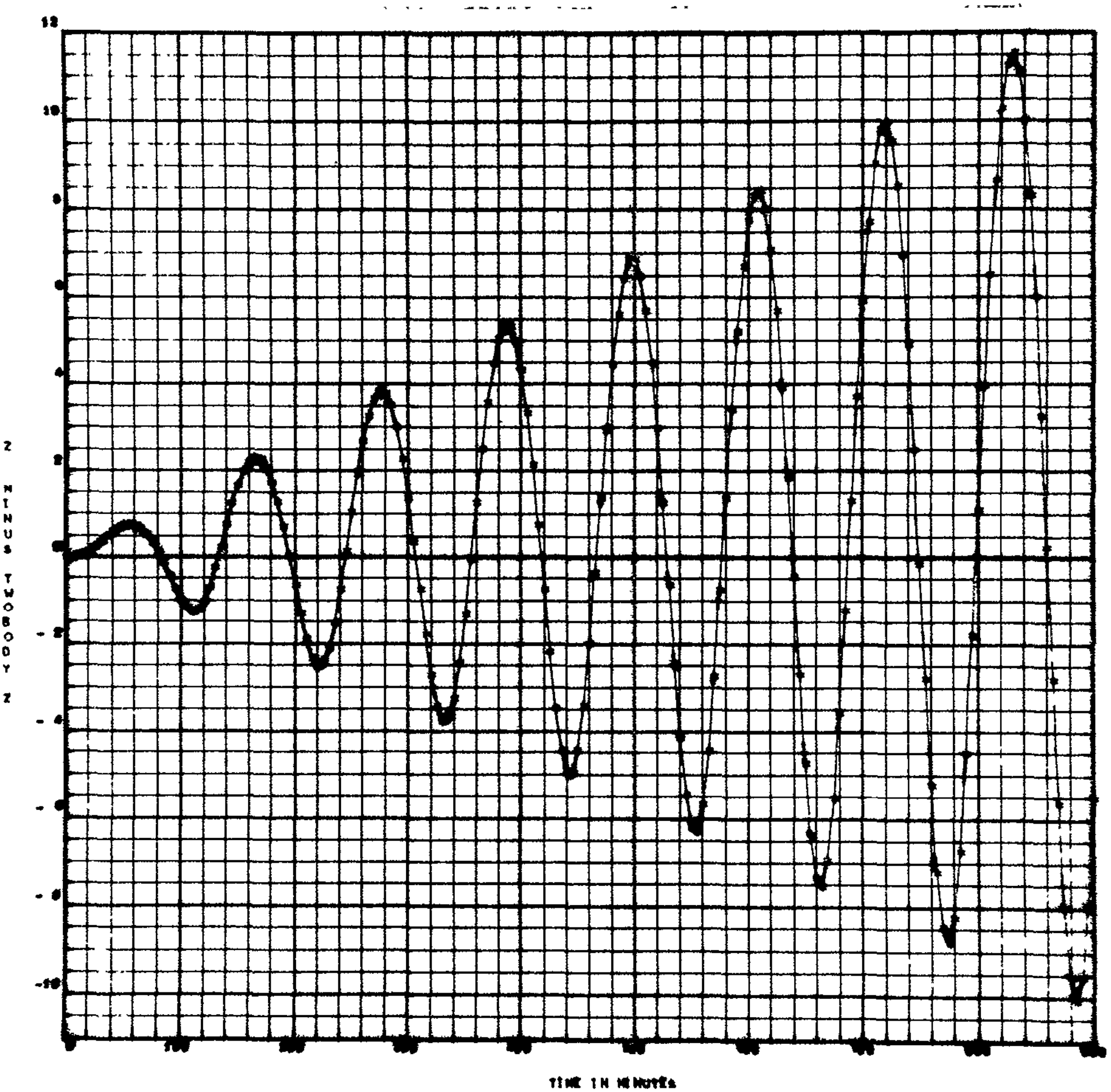

Fig. 10 – Deviation in Modules Z-Relative Position from Two-Body Relative Z Position (meters) vs. Time (min)

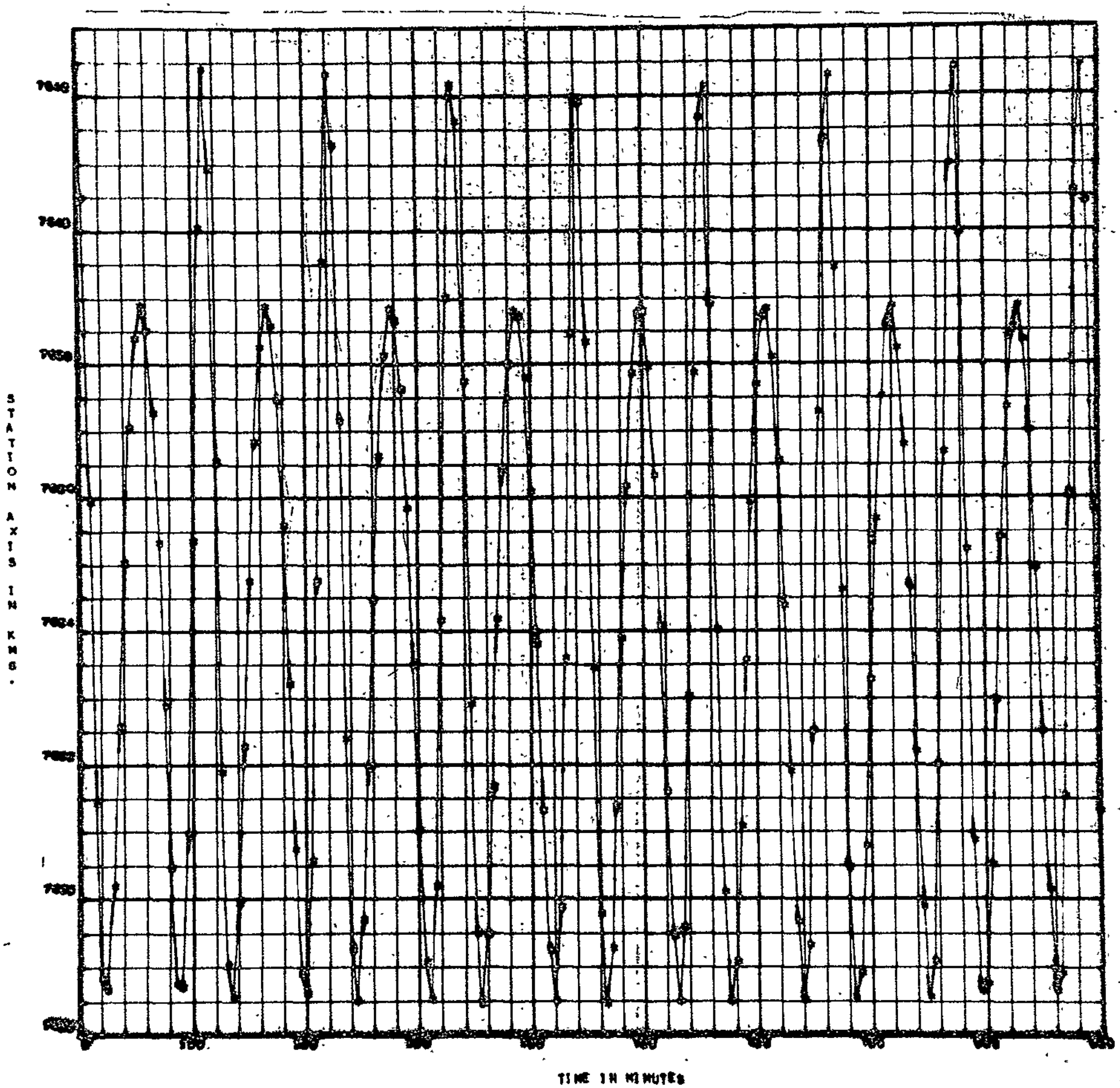


Fig. 11 — Stations Semi-Major Axis (km) vs. Time (min)


Fig. 12 — Stations Eccentricity vs. Time (min)

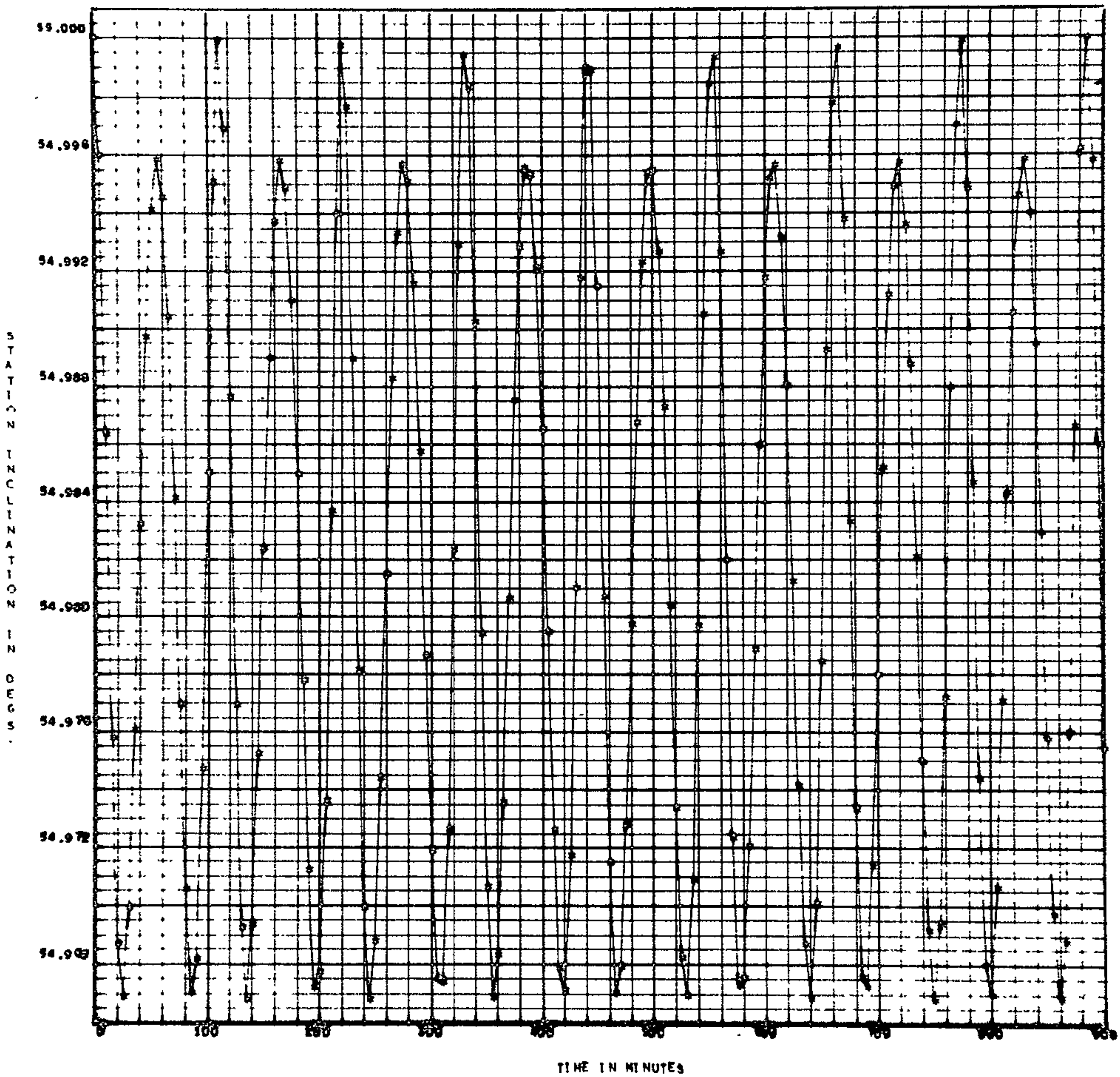


Fig. 13 — Stations Inclination (deg) vs. Time (min)

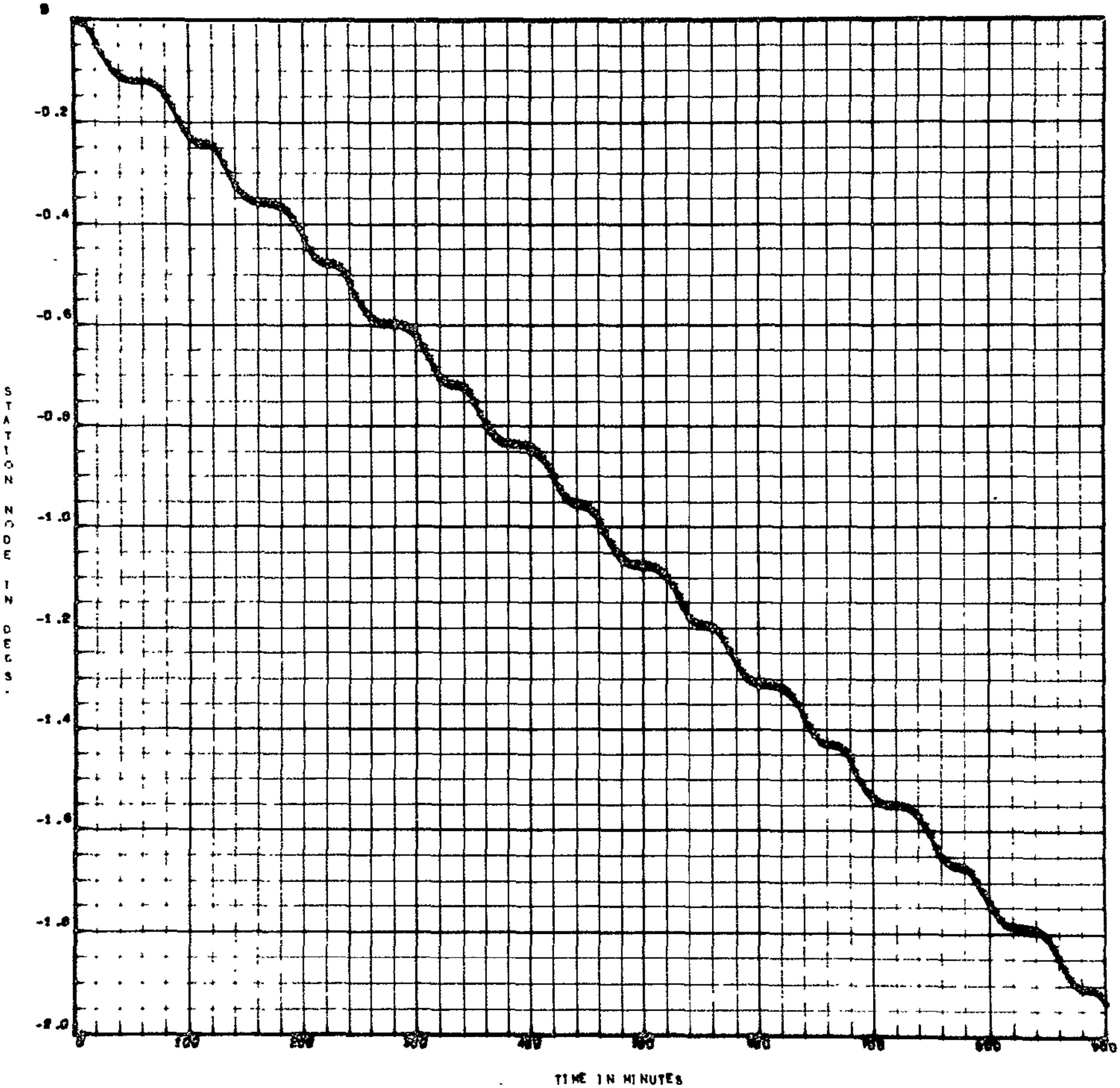


Fig. 14 – Stations Ascending Node (deg) vs. Time (min)

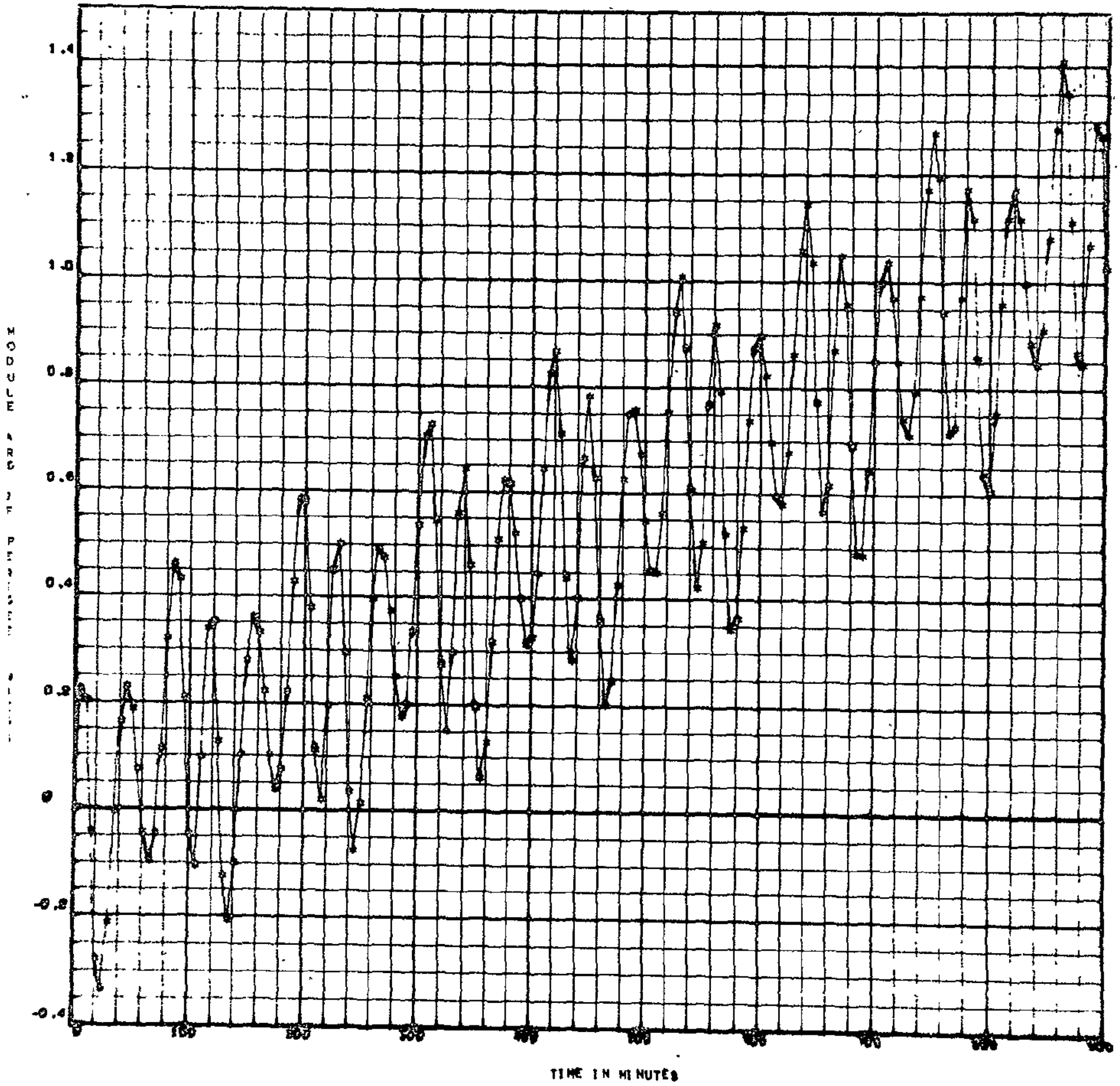


Fig. 15 – Stations Argument of Perigee (deg) vs. Time (min)

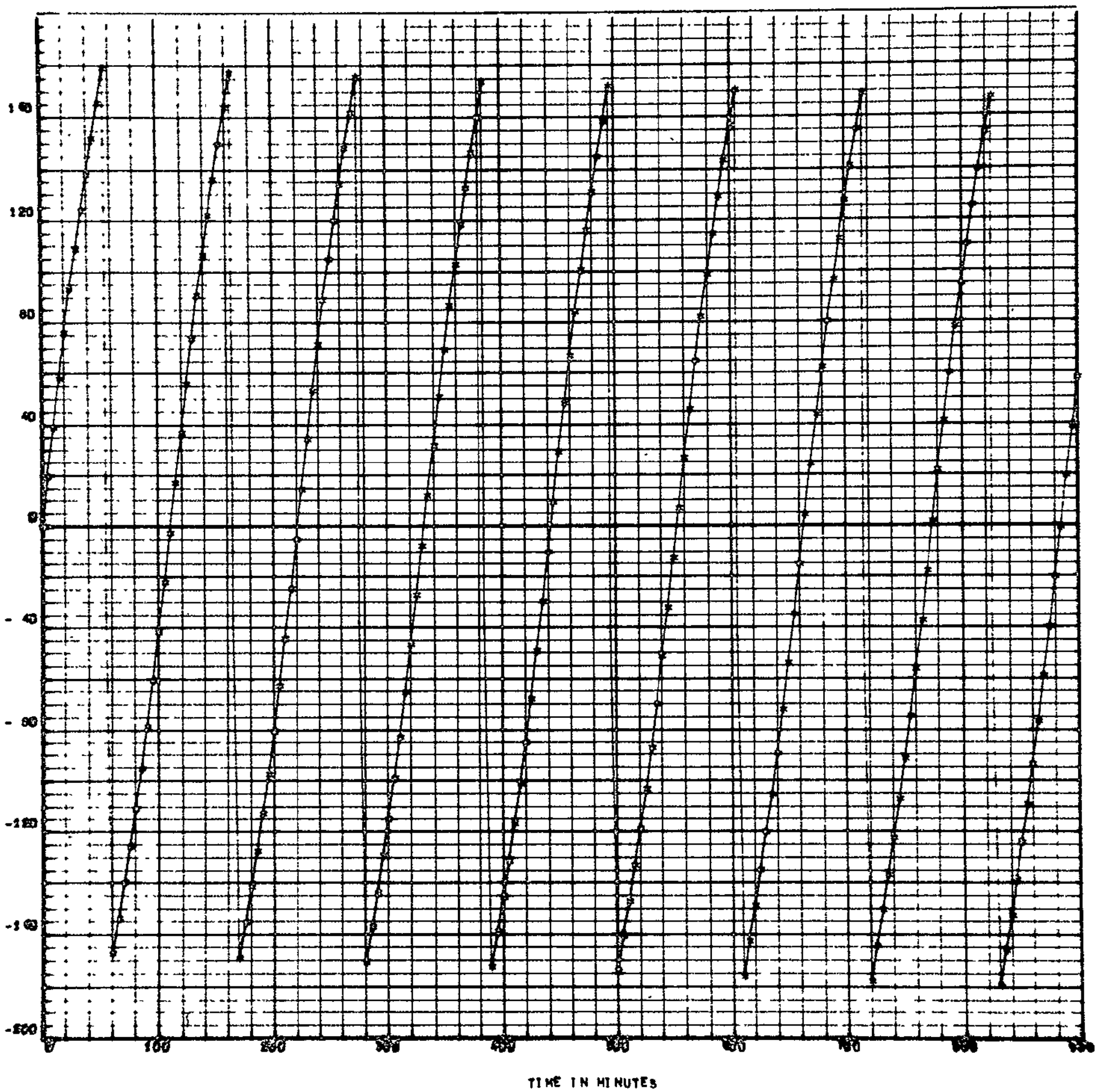


Fig. 16 – Stations True Anomaly (deg) vs. Time (min)

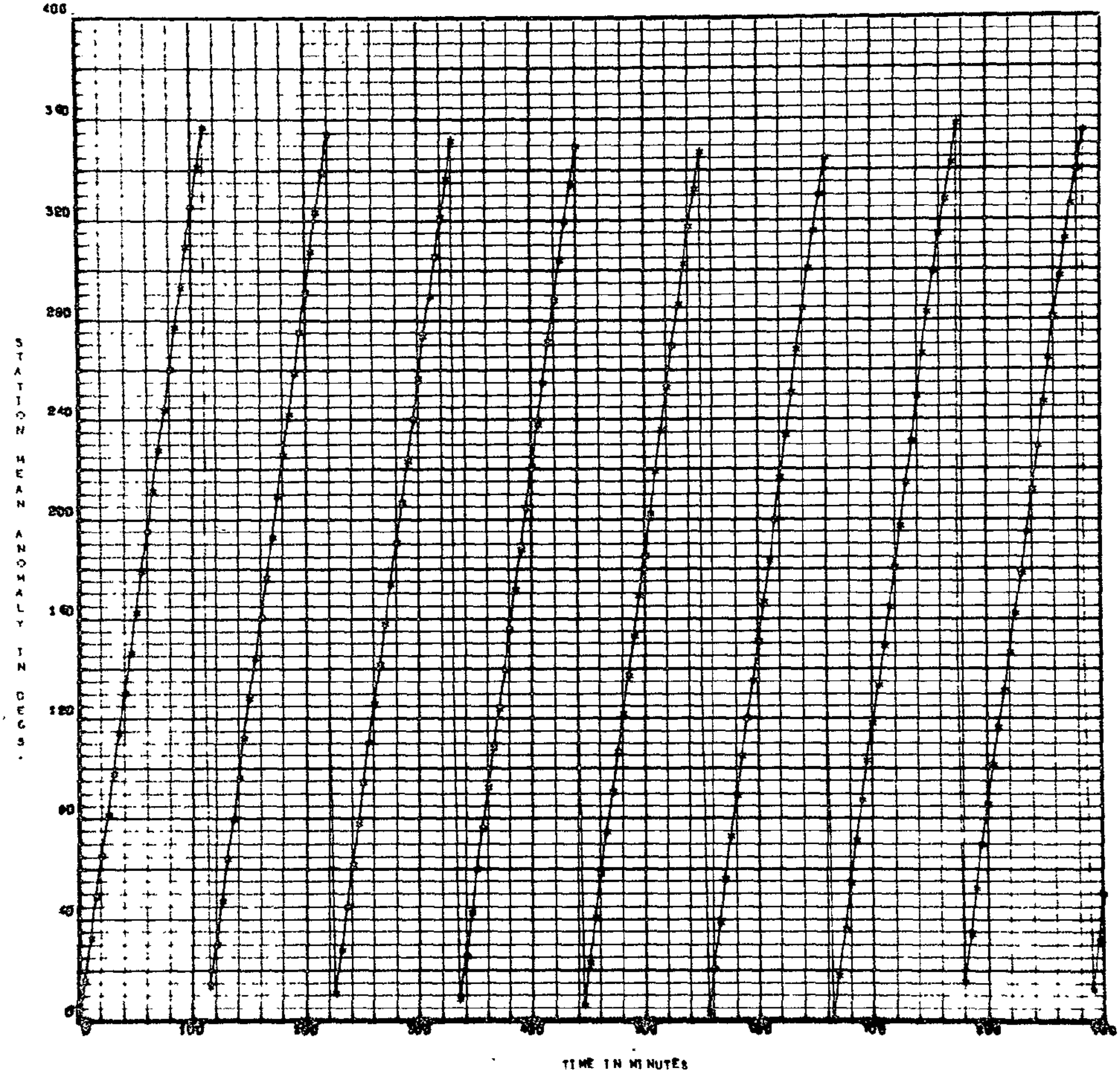


Fig. 17 — Stations Mean Anomaly (deg) vs. Time (min)

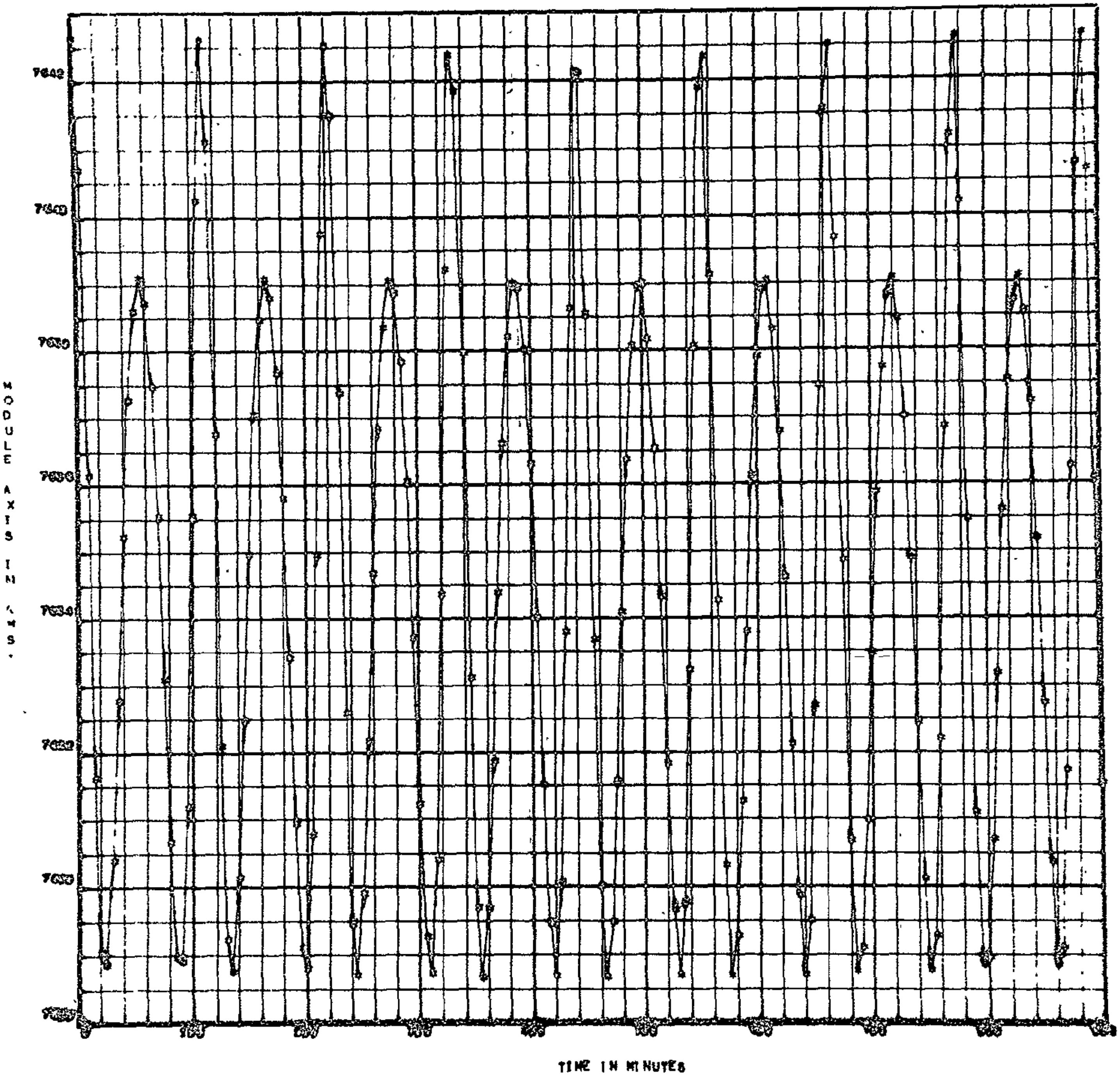

Fig. 18 – Modules Semi-Major Axis (km) vs. Time (min)


Fig. 19 – Modules Eccentricity vs. Time (min)

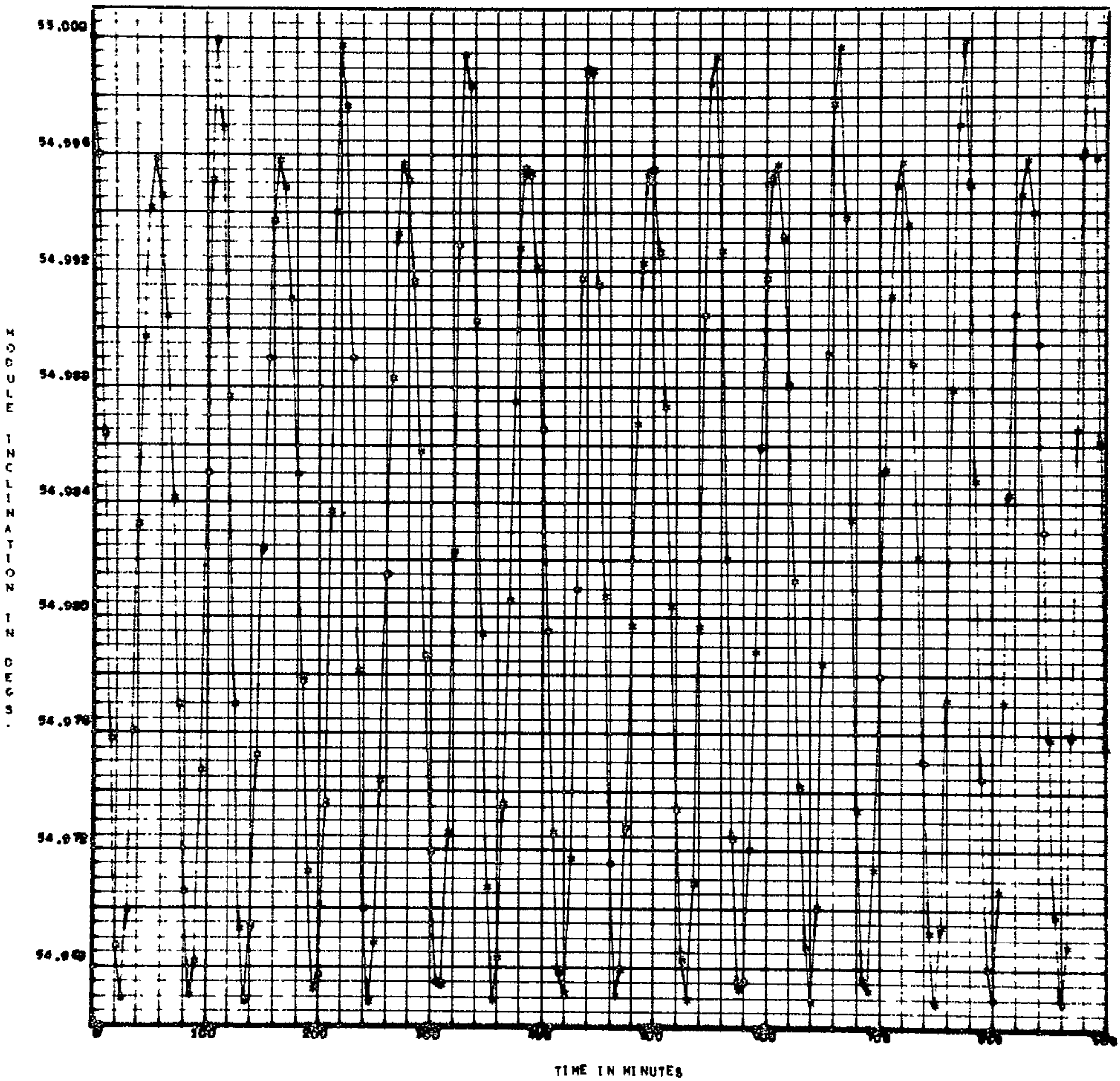


Fig. 20 — Modules Inclination (deg) vs. Time (min)

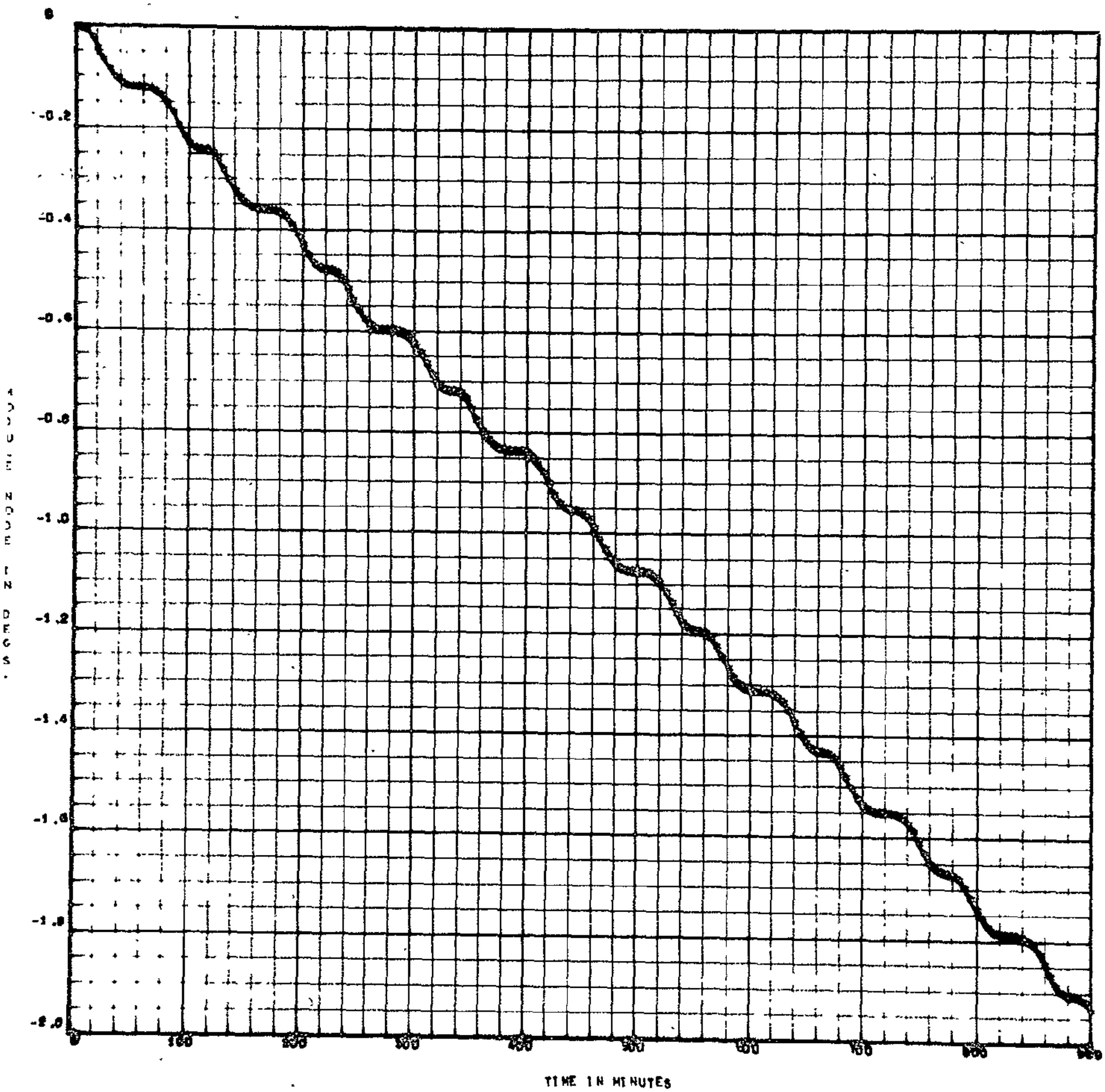


Fig. 21 – Modules Ascending Node (deg) vs. Time (min)

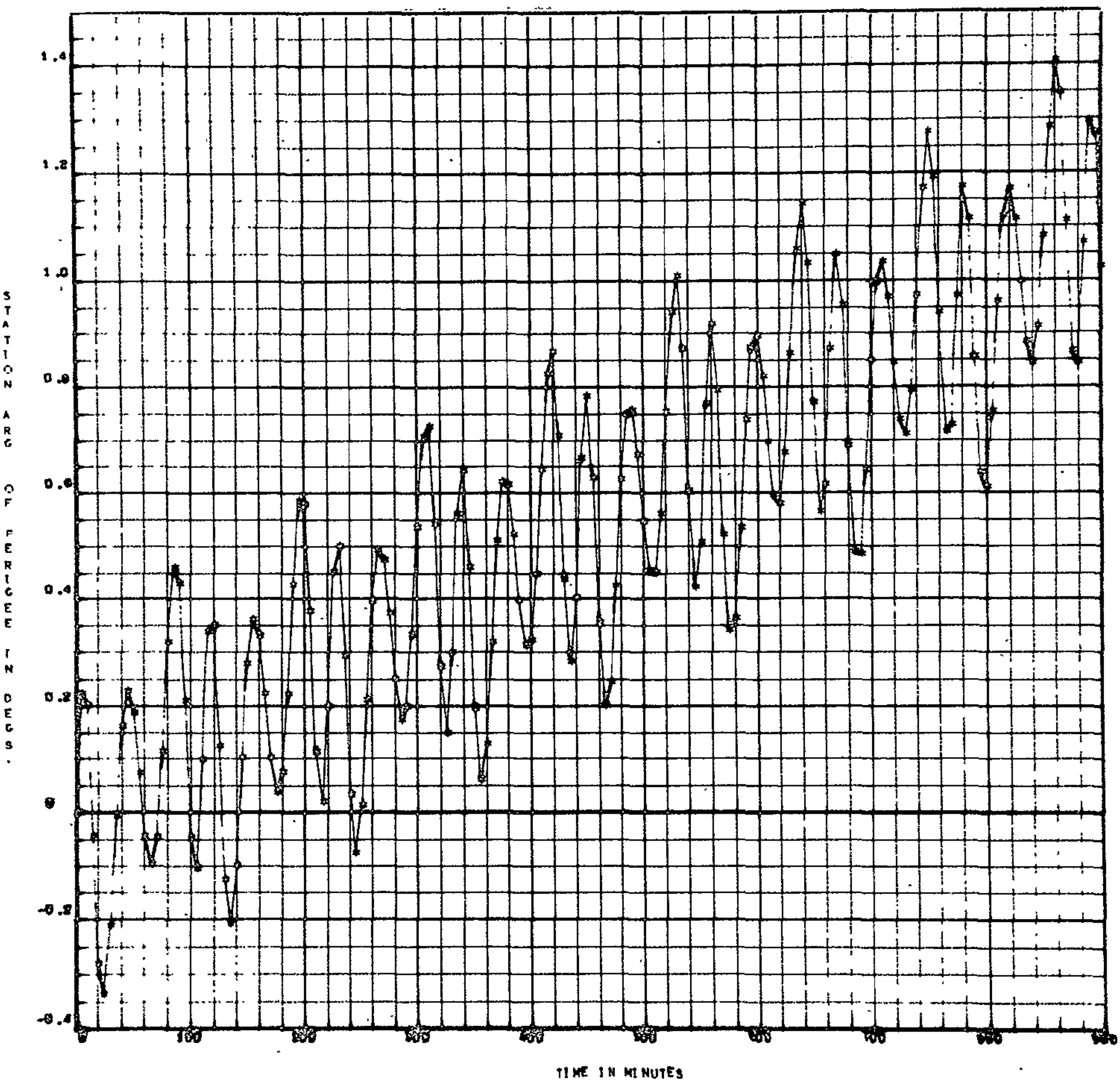


Fig. 22 – Modules Argument of Perigee (deg) vs. Time (min)

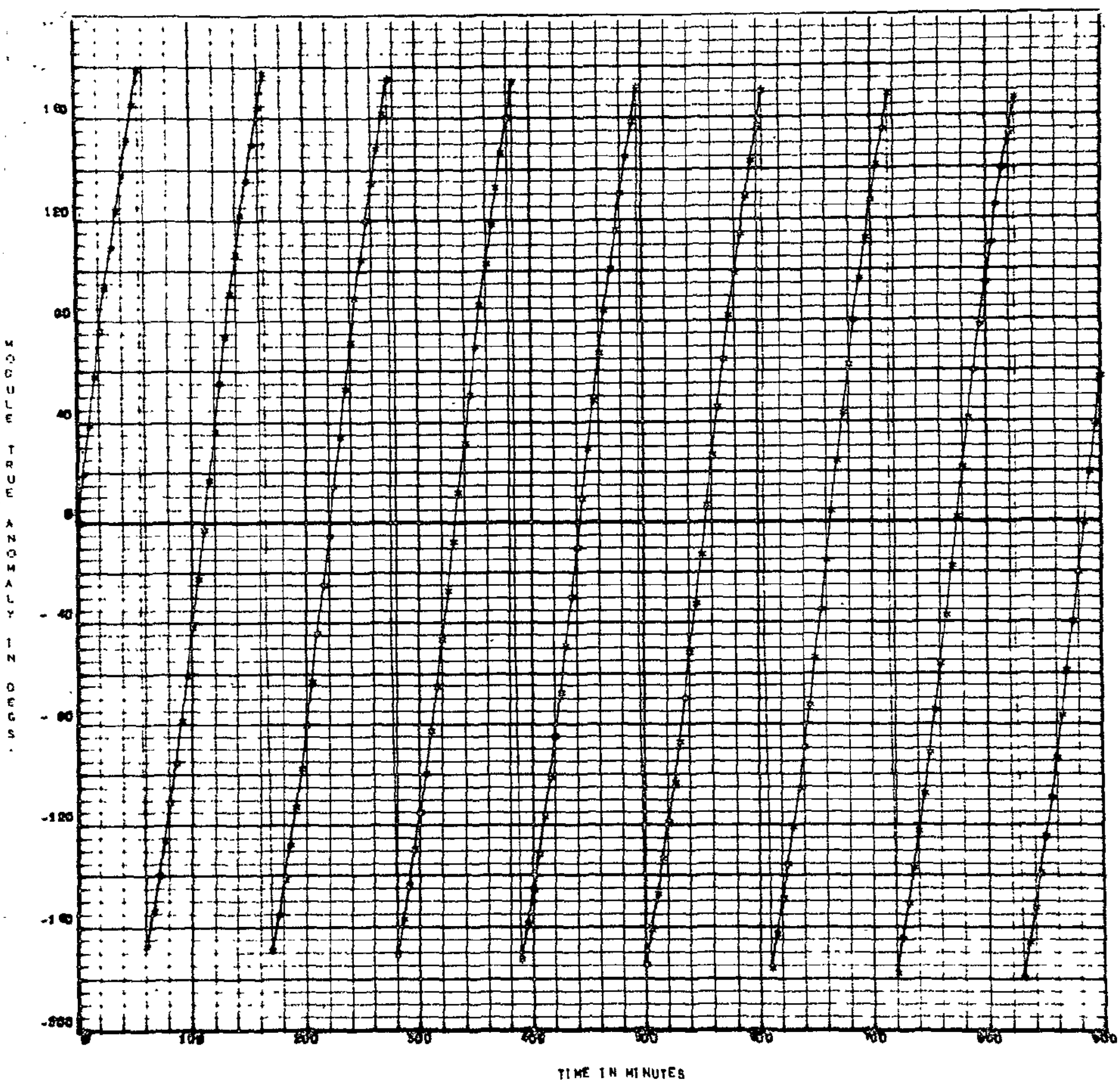


Fig. 23 – Modules True Anomaly (deg) vs. Time (min)

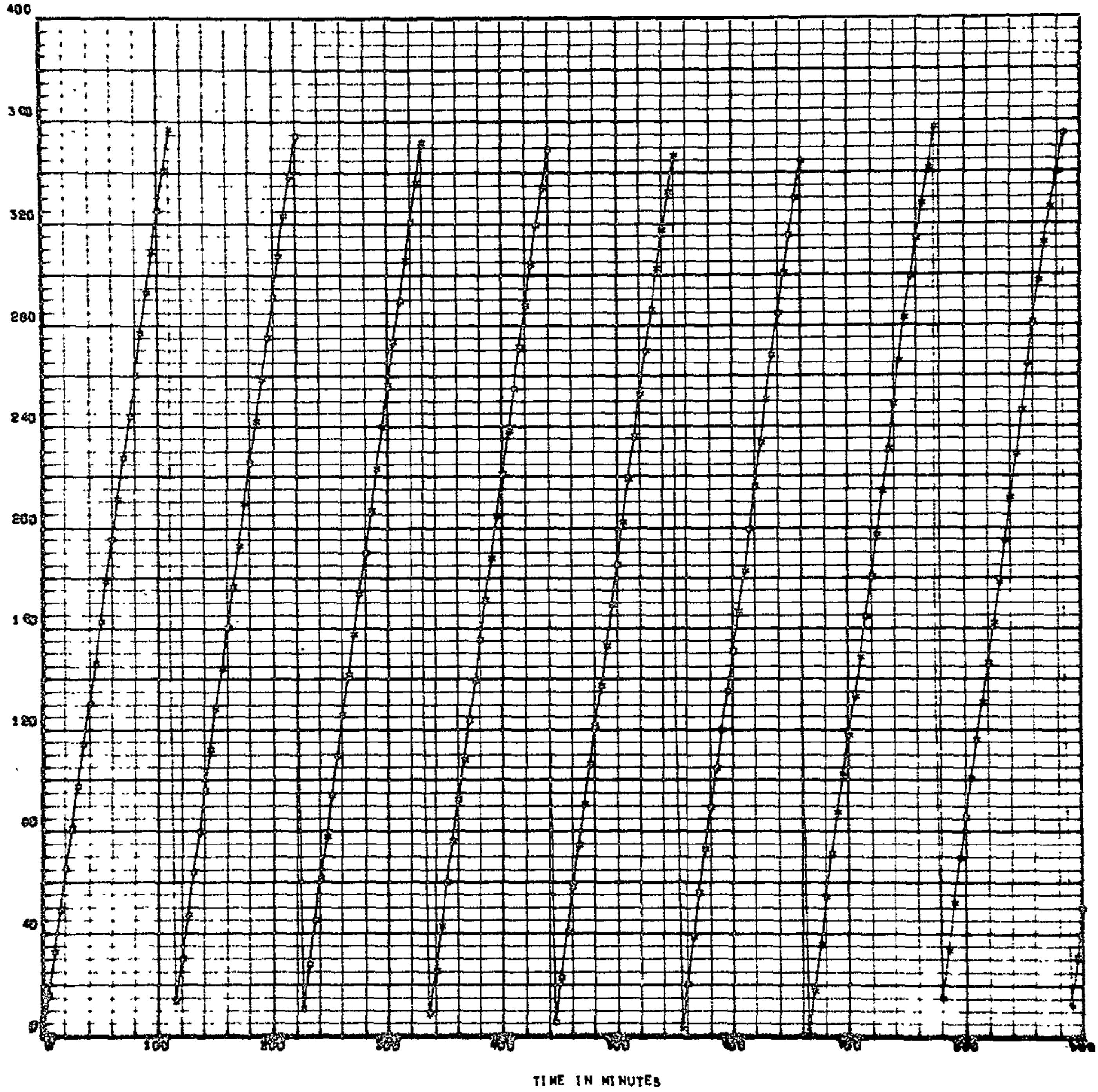


Fig. 24 – Modules Mean Anomaly (deg) vs. Time (min)

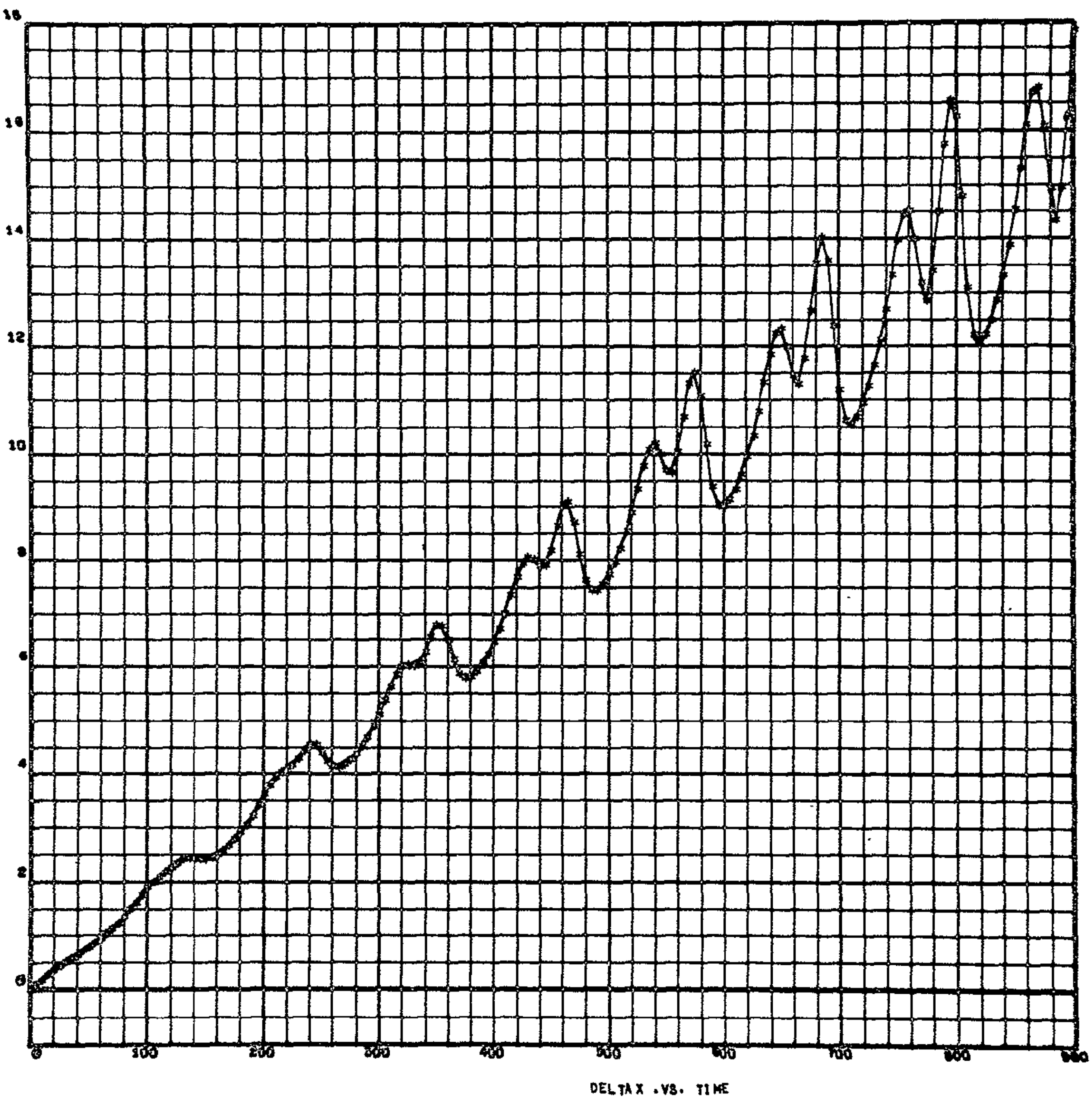


Fig. 25 — Modules X-Relative Position (km) vs. Time (min)
(Koelle's Equations)

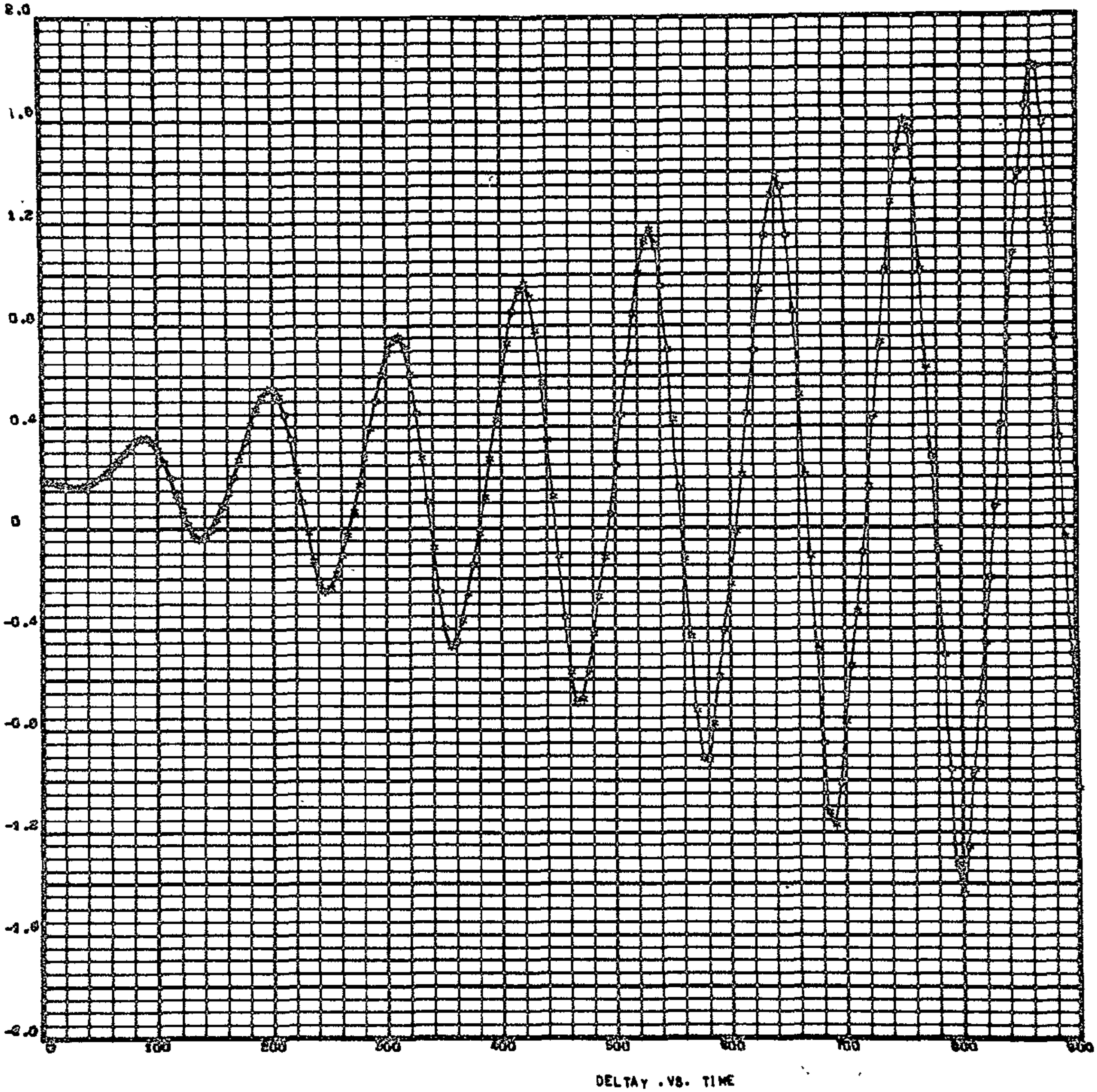


Fig. 26 — Modules Y-Relative Position (km) vs. Time (min)
(Koelle's Equations)

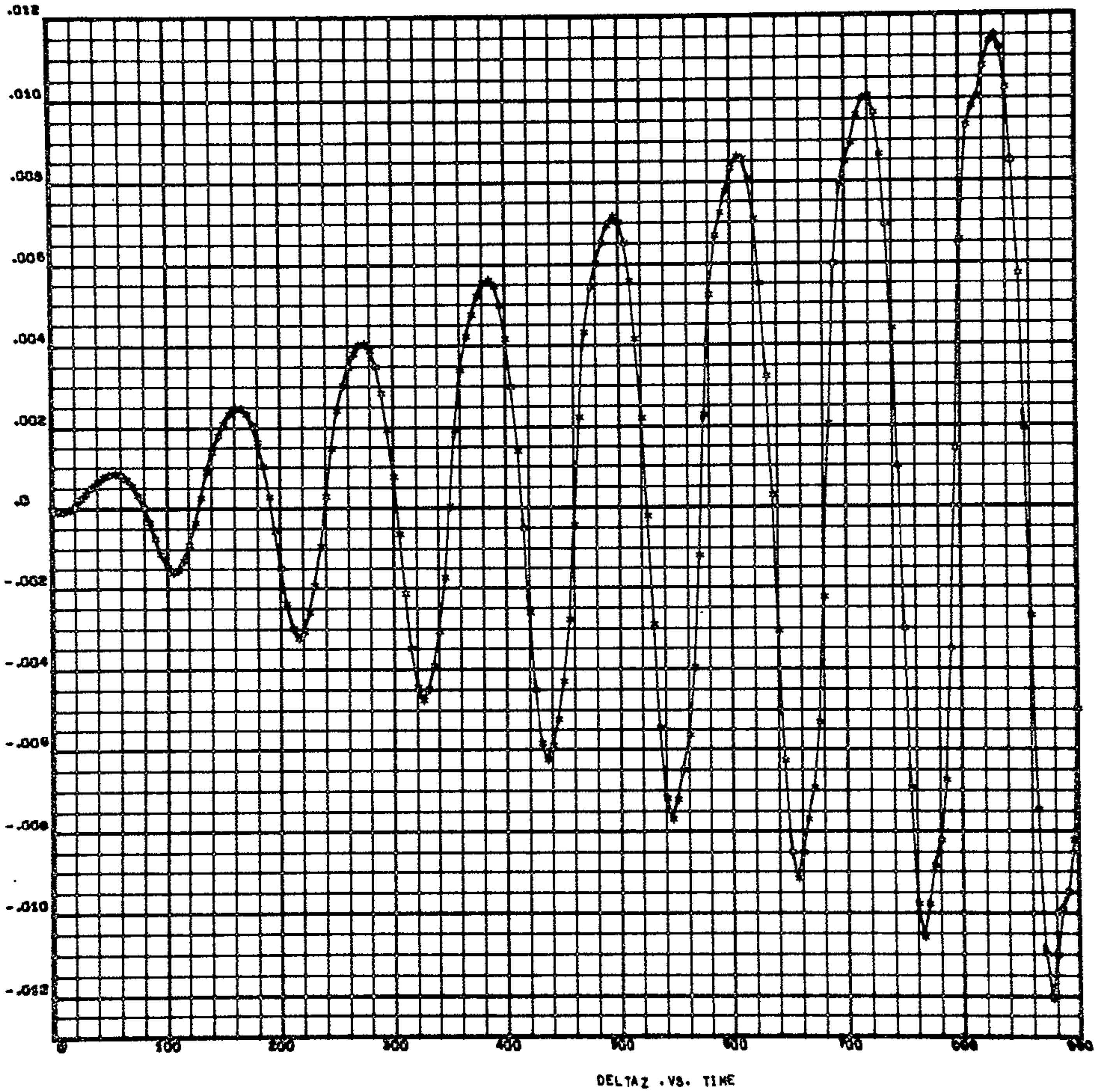


Fig. 27 – Modules Z-Relative Position (km) vs. Time (min)
(Koelle's Equations)

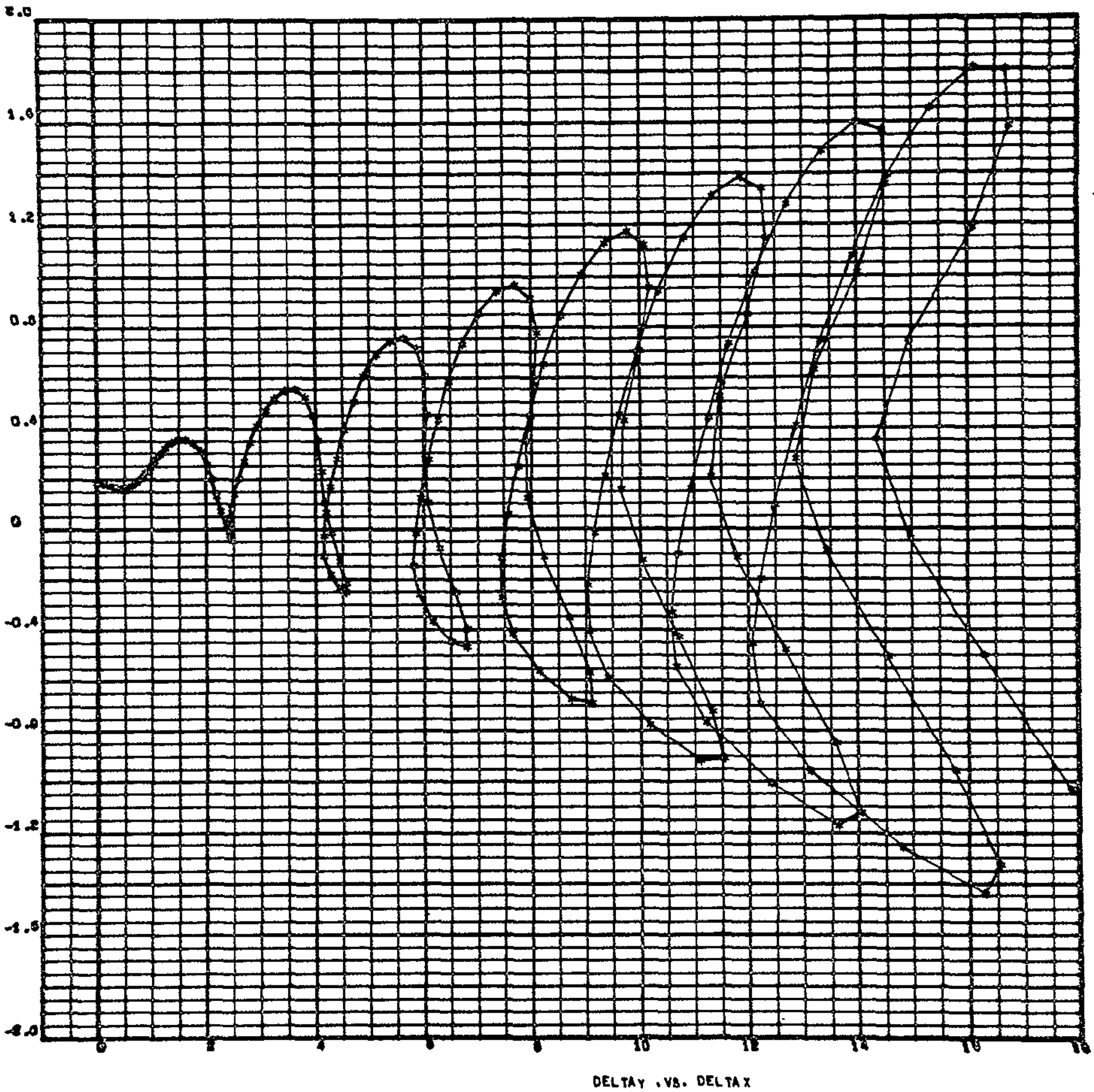


Fig. 28 – Modules Y-Relative Position (km) vs. Modules X-Relative Position (km) – Motion in Stations Plane (Koelles Equations)

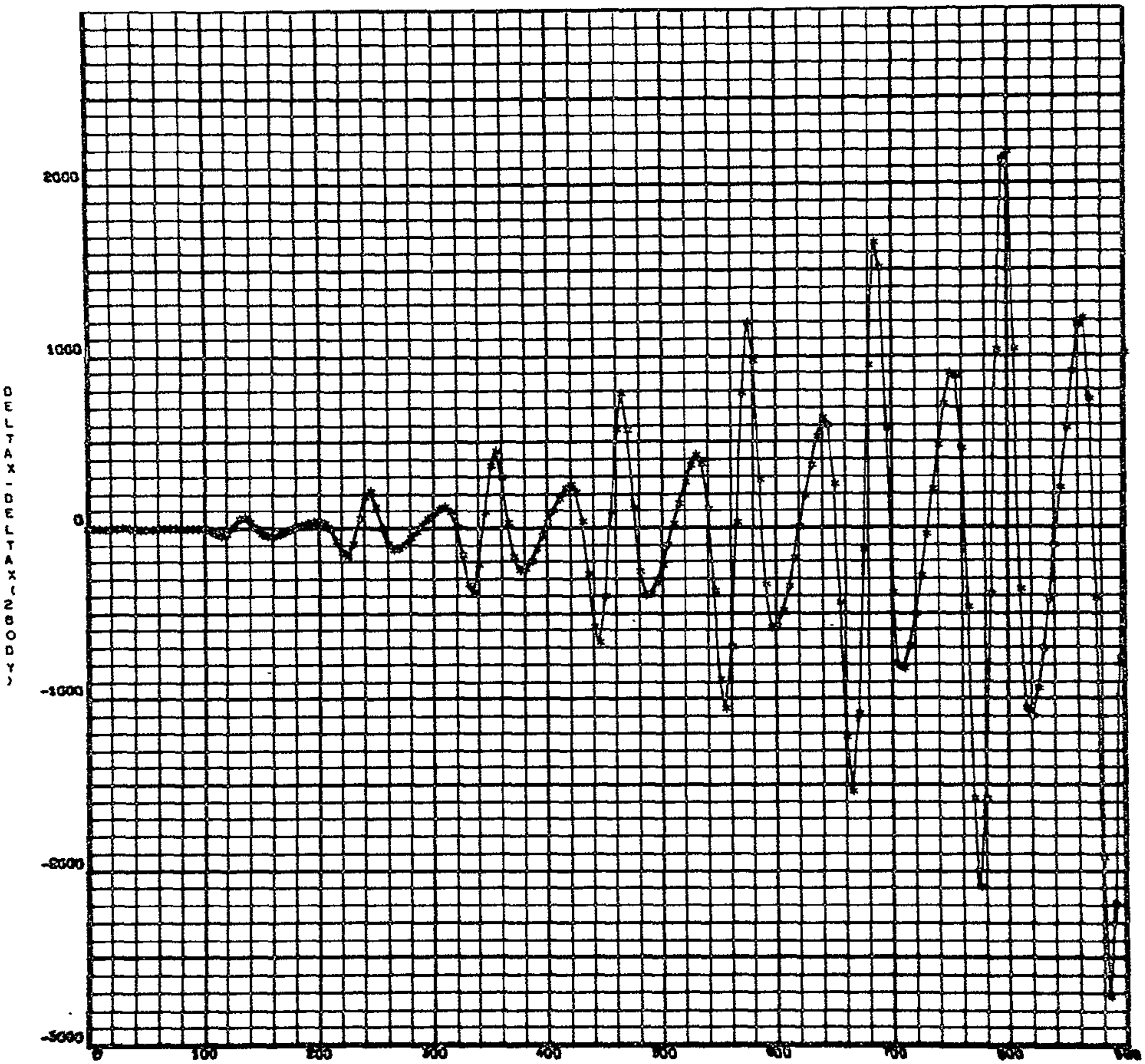


Fig. 29 — Deviation in Modules X-Relative Position from Two-Body Relative X-Position (meters) vs. Time (min) (Koelle's Equations)

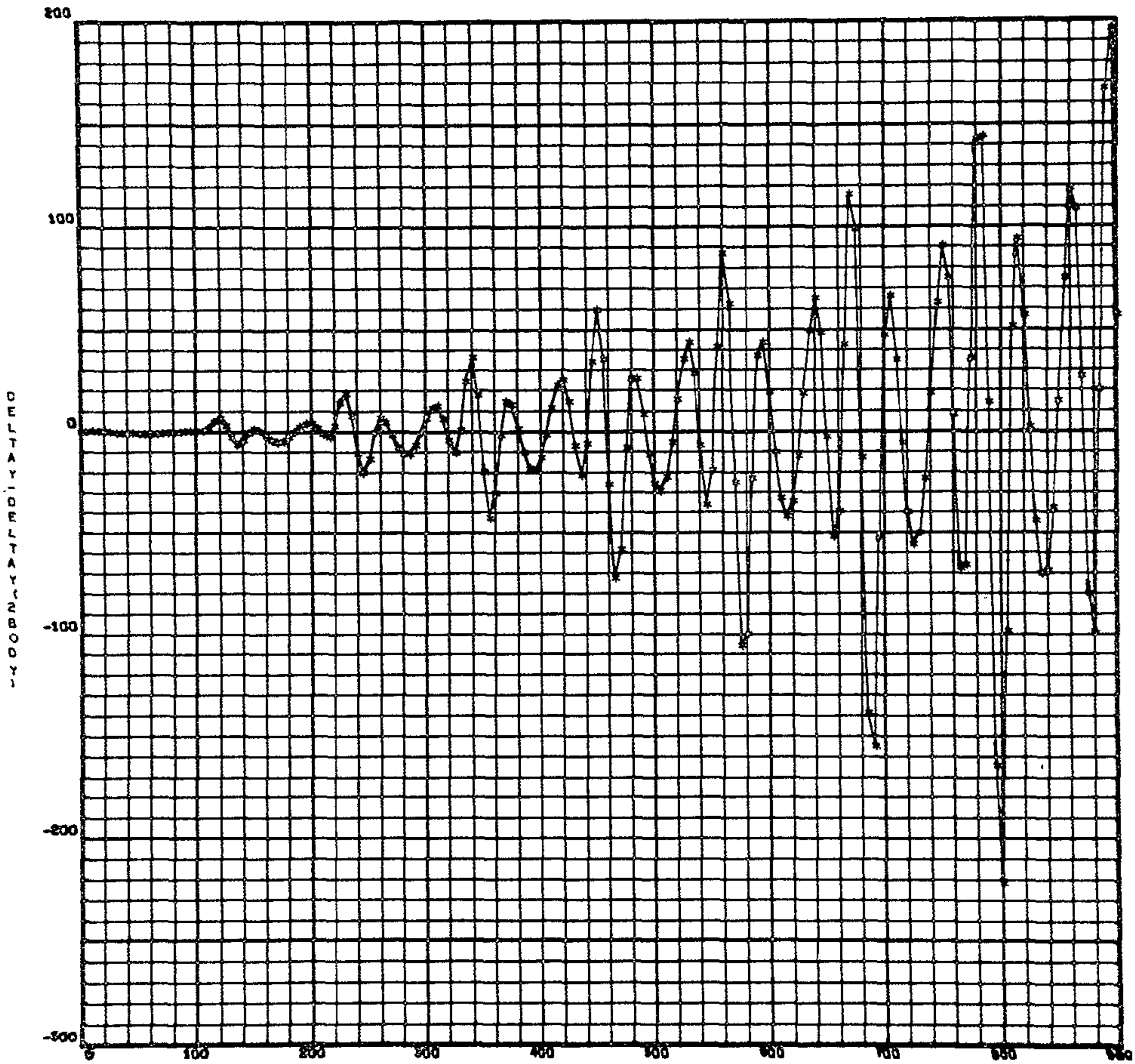


Fig. 30 – Deviation in Modules Y-Relative Position from Two-Body Relative Y Position (meters) vs. Time (min) (Koelle's Equations)

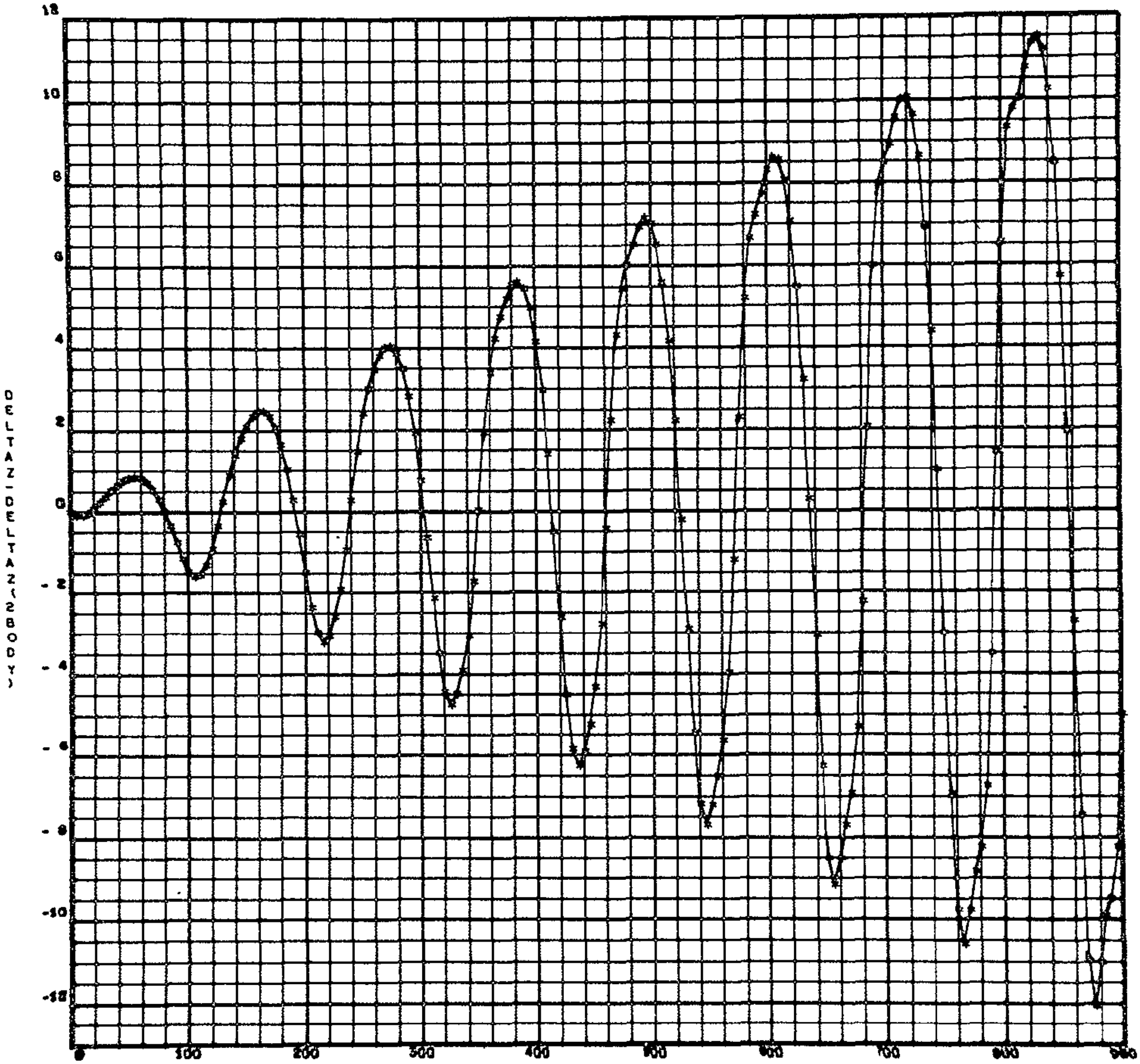


Fig. 31 – Deviation in Modules Z-Relative Position from Two-Body Relative Z Position (meters) vs. Time (min) (Koelle's Equations)

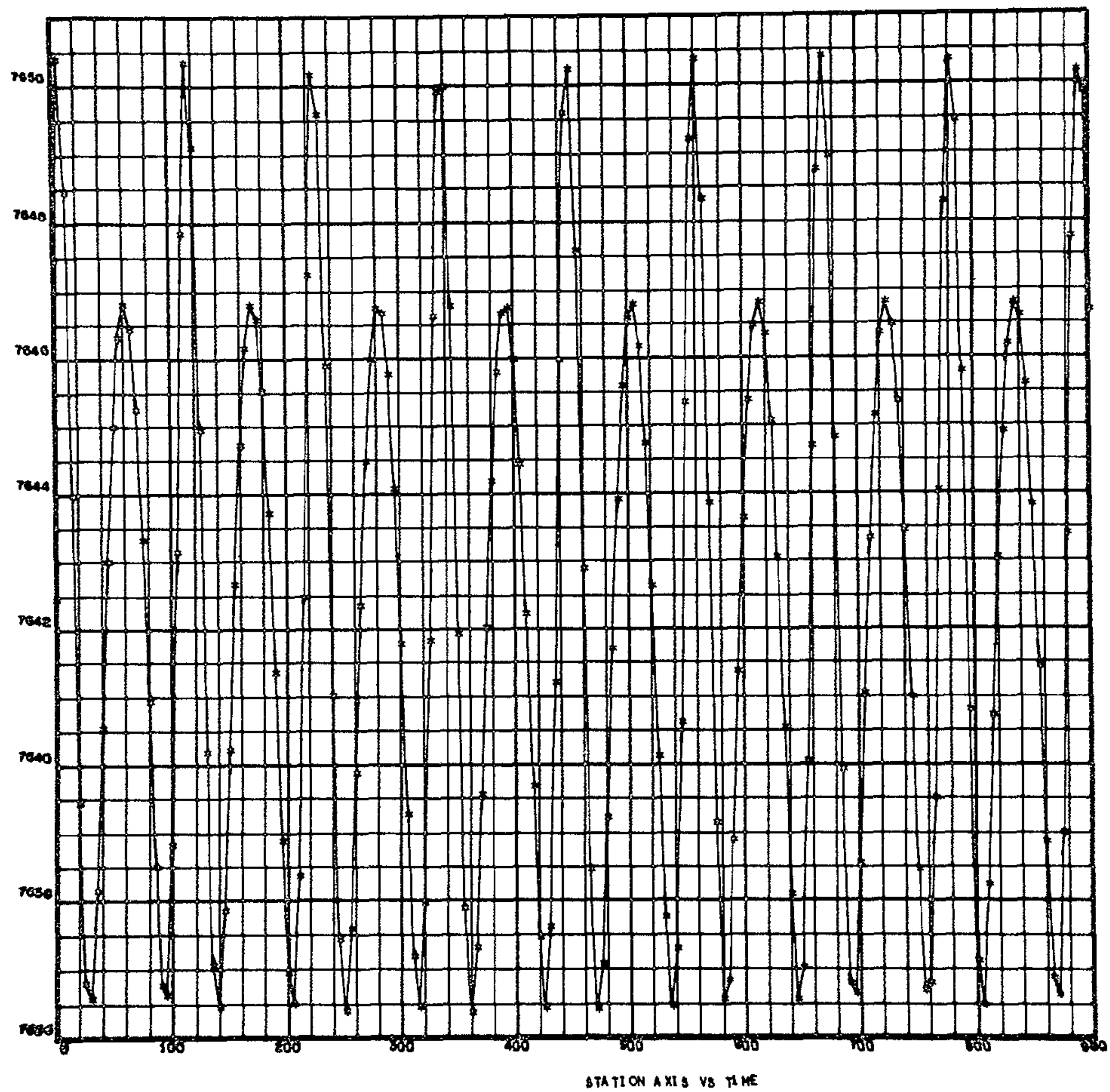


Fig. 32 – Stations Semi-Major Axis (km) vs. Time (min)
(Koelle's Equations)

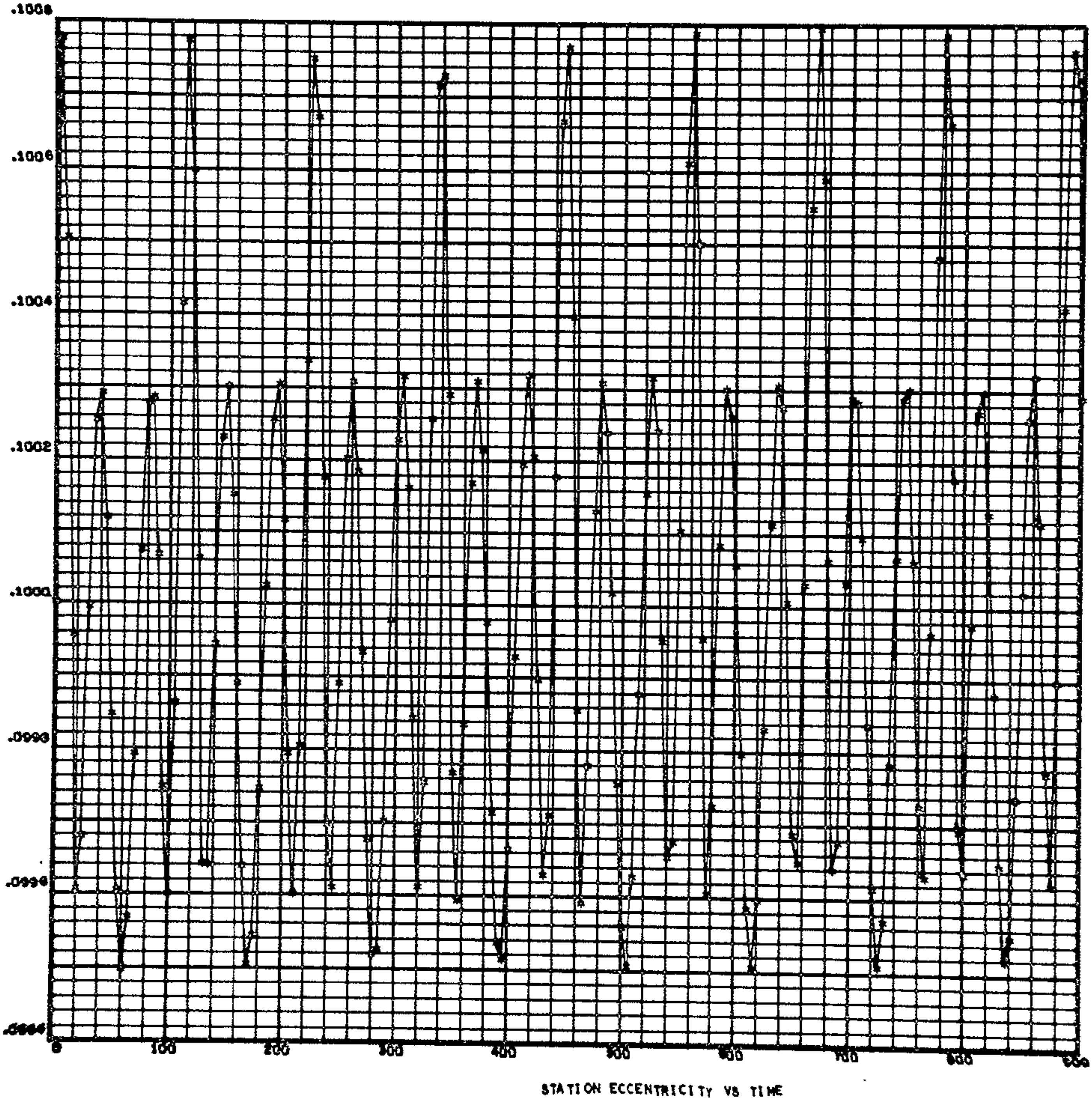


Fig. 33 — Stations Eccentricity vs. Time (min)
(Koelle's Equations)

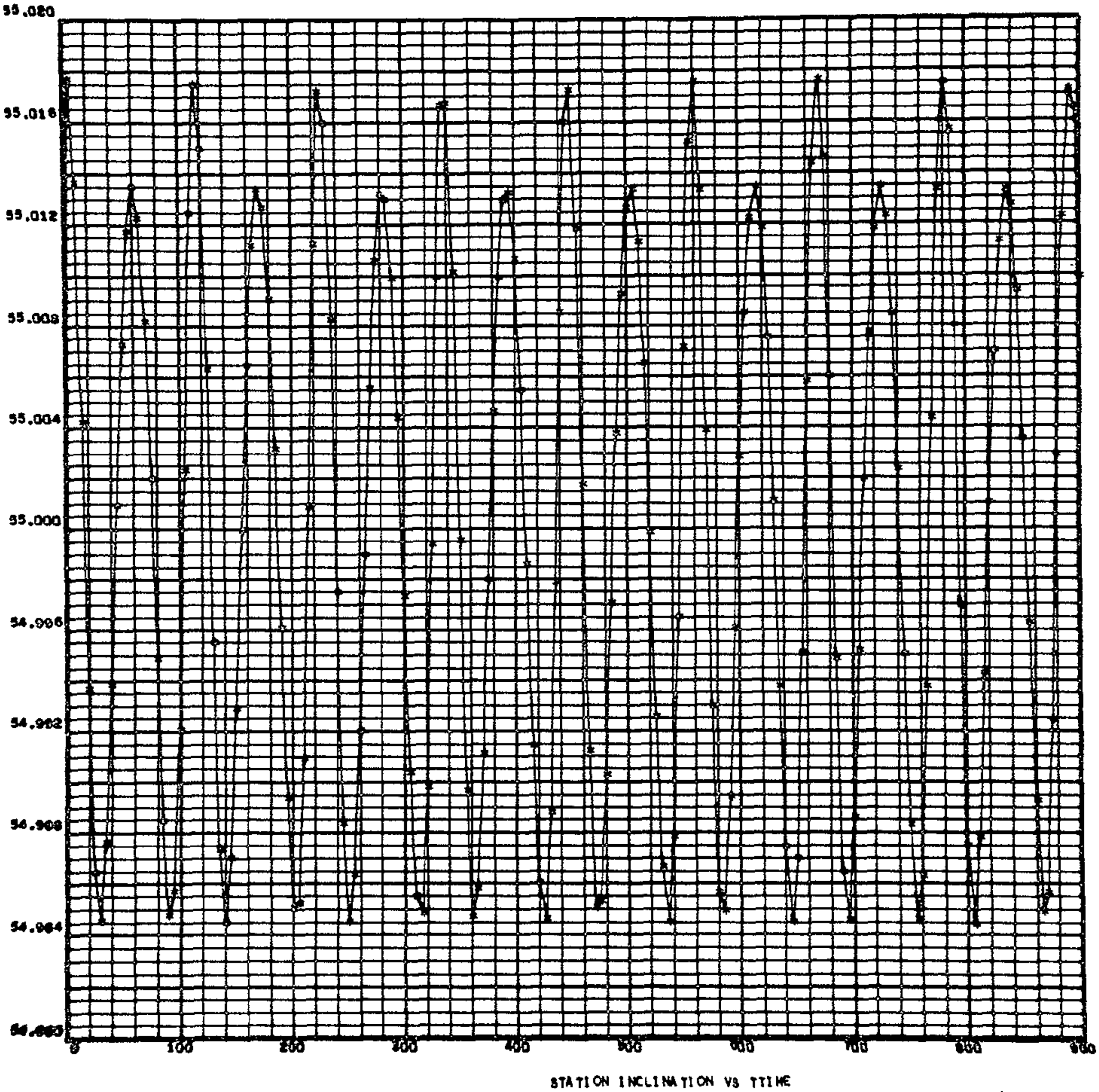


Fig. 34 – Stations Inclination (deg) vs. Time (min)
(Koelle's Equations)

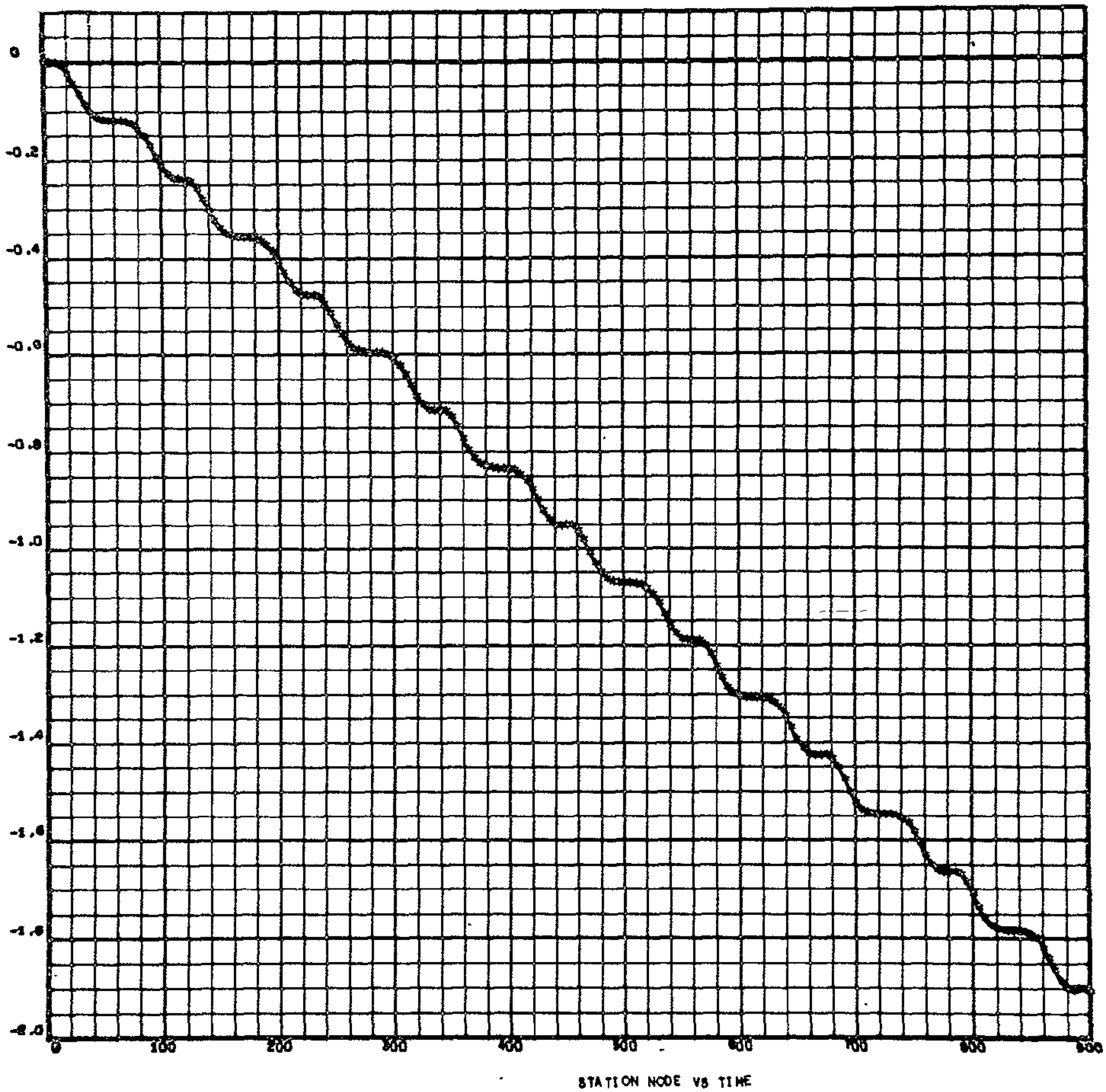


Fig. 35 – Stations Ascending Node (deg) vs. Time (min)
(Koelle's Equations)

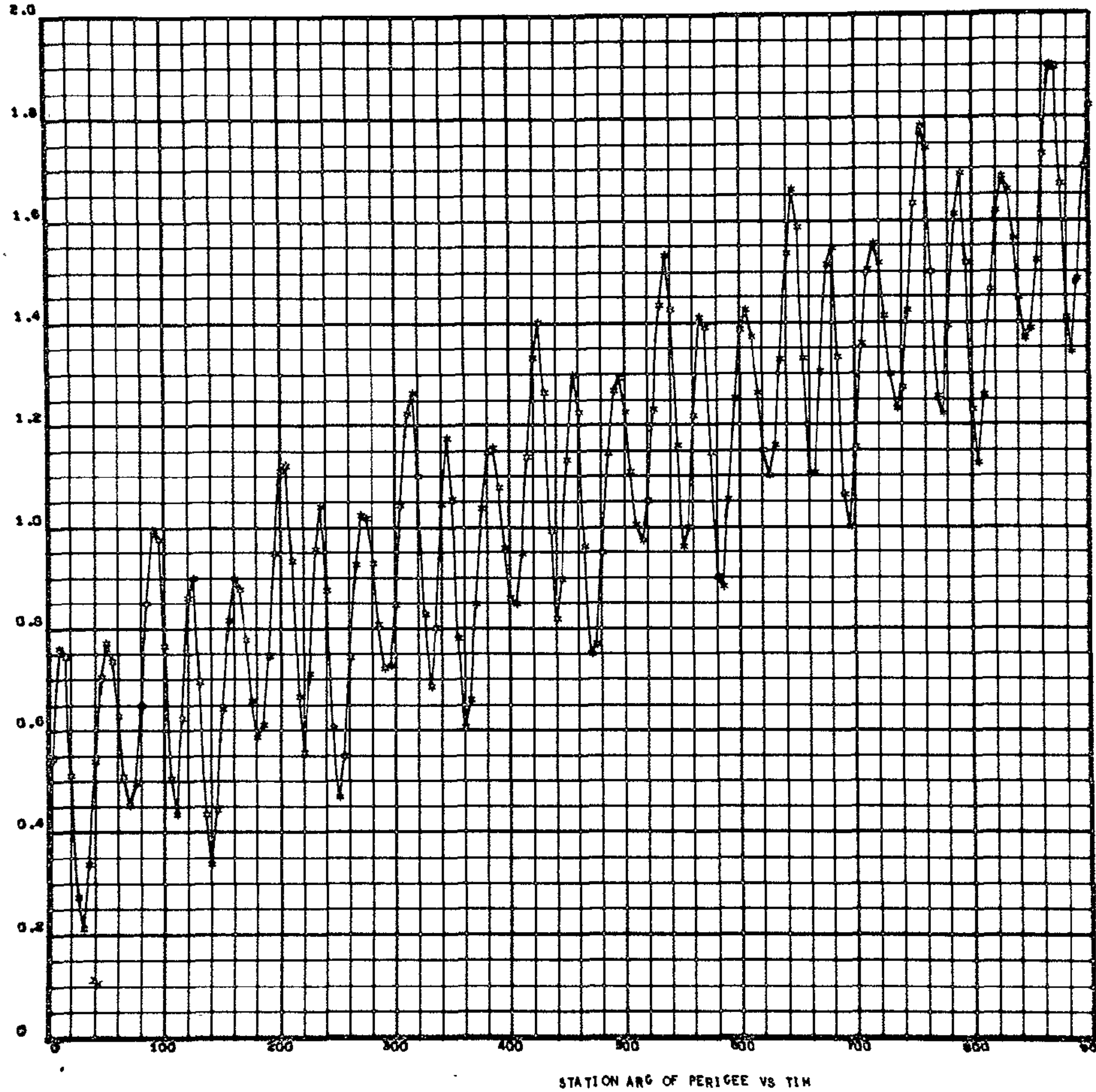


Fig. 36 – Stations Argument of Perigee (deg) vs. Time (min)
(Koelle's Equations)

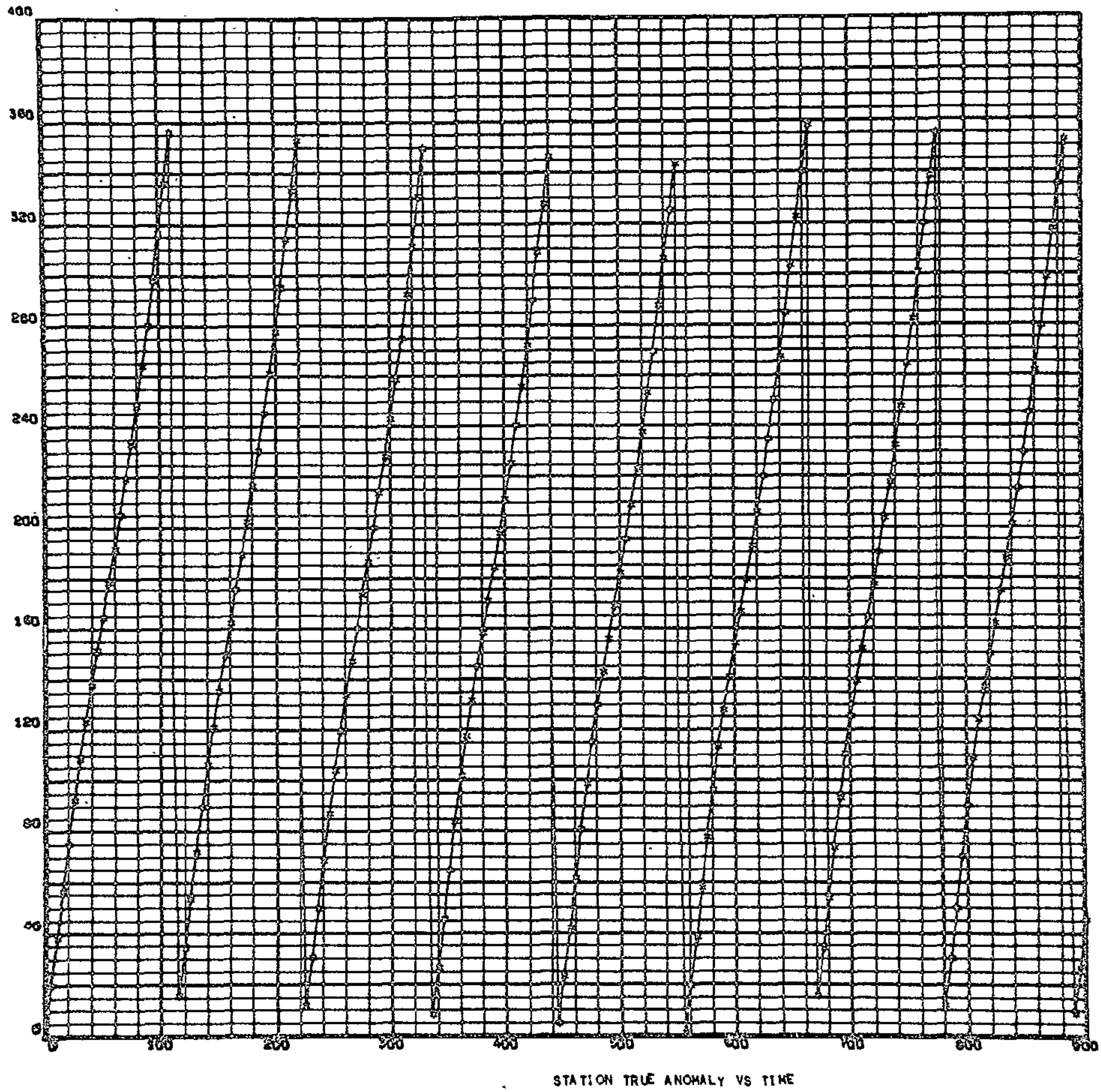


Fig. 37 — Stations True Anomaly (deg) vs. Time (min)
(Koelle's Equations)

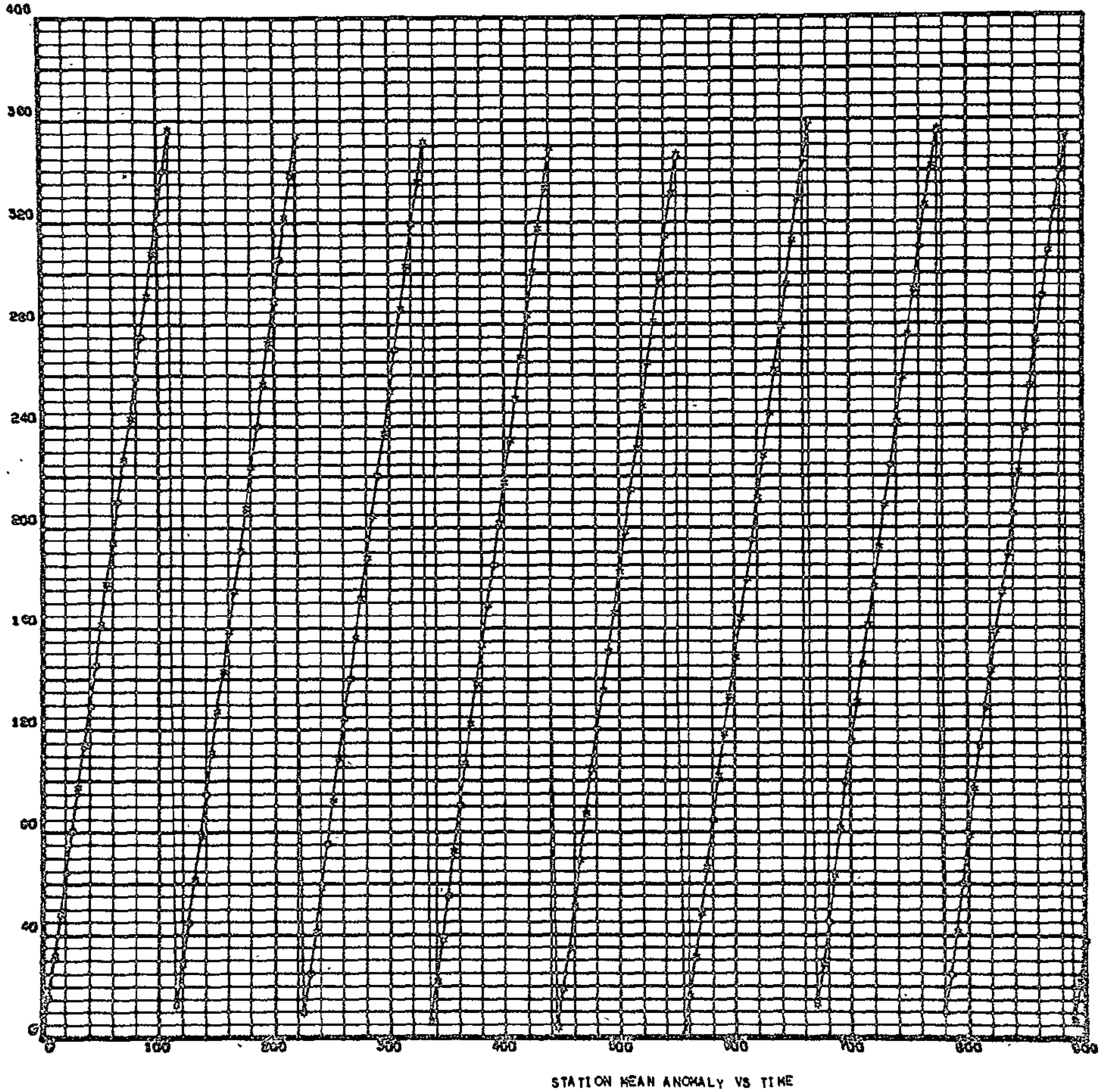


Fig. 38 – Stations Mean Anomaly (deg) vs. Time (sec)
(Koelle's Equations)

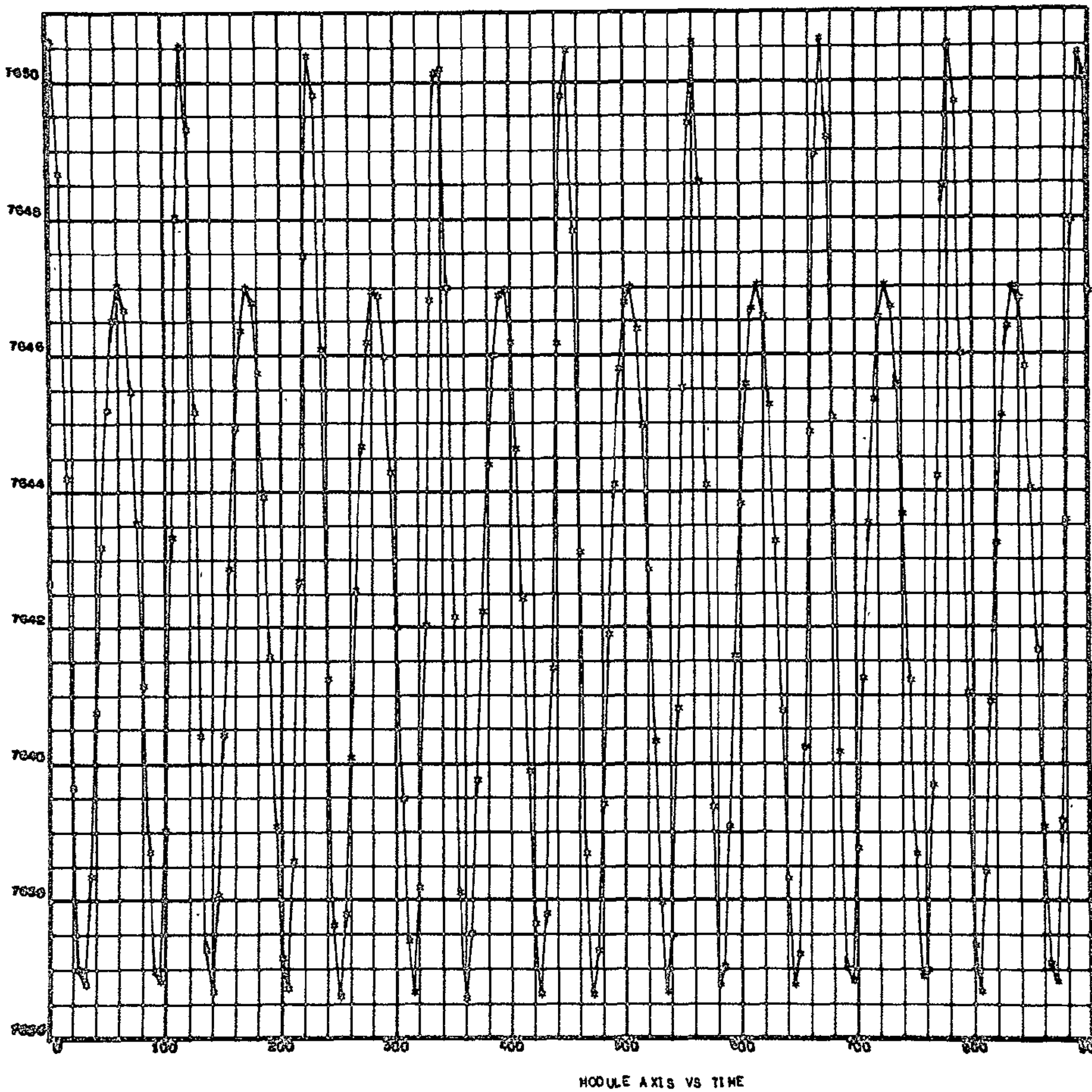


Fig. 39 – Modules Semi-Major Axis (km) vs. Time (min)
(Koelle's Equations)

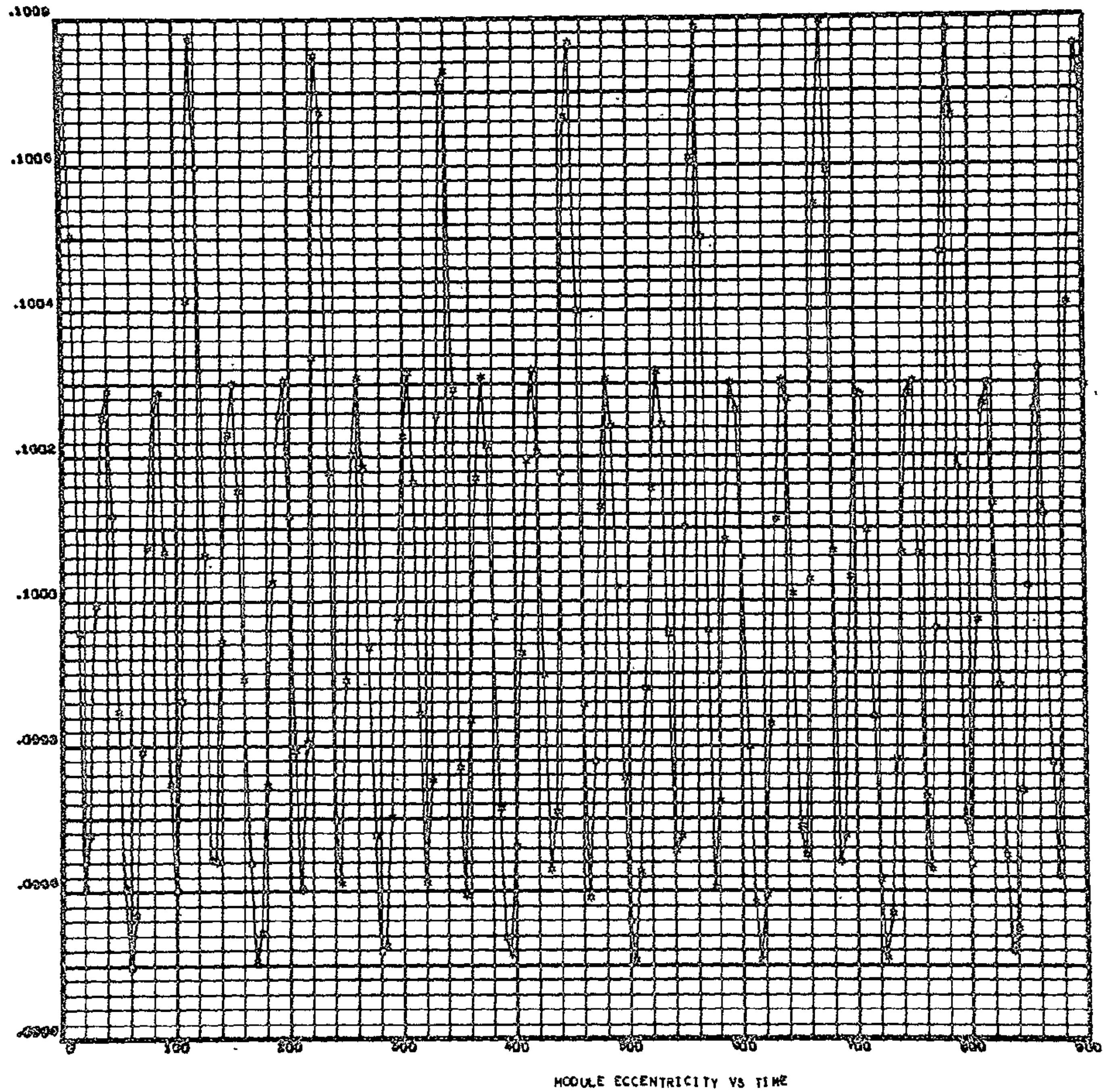


Fig. 40 – Modules Eccentricity vs. Time (min)
(Koelle's Equations)

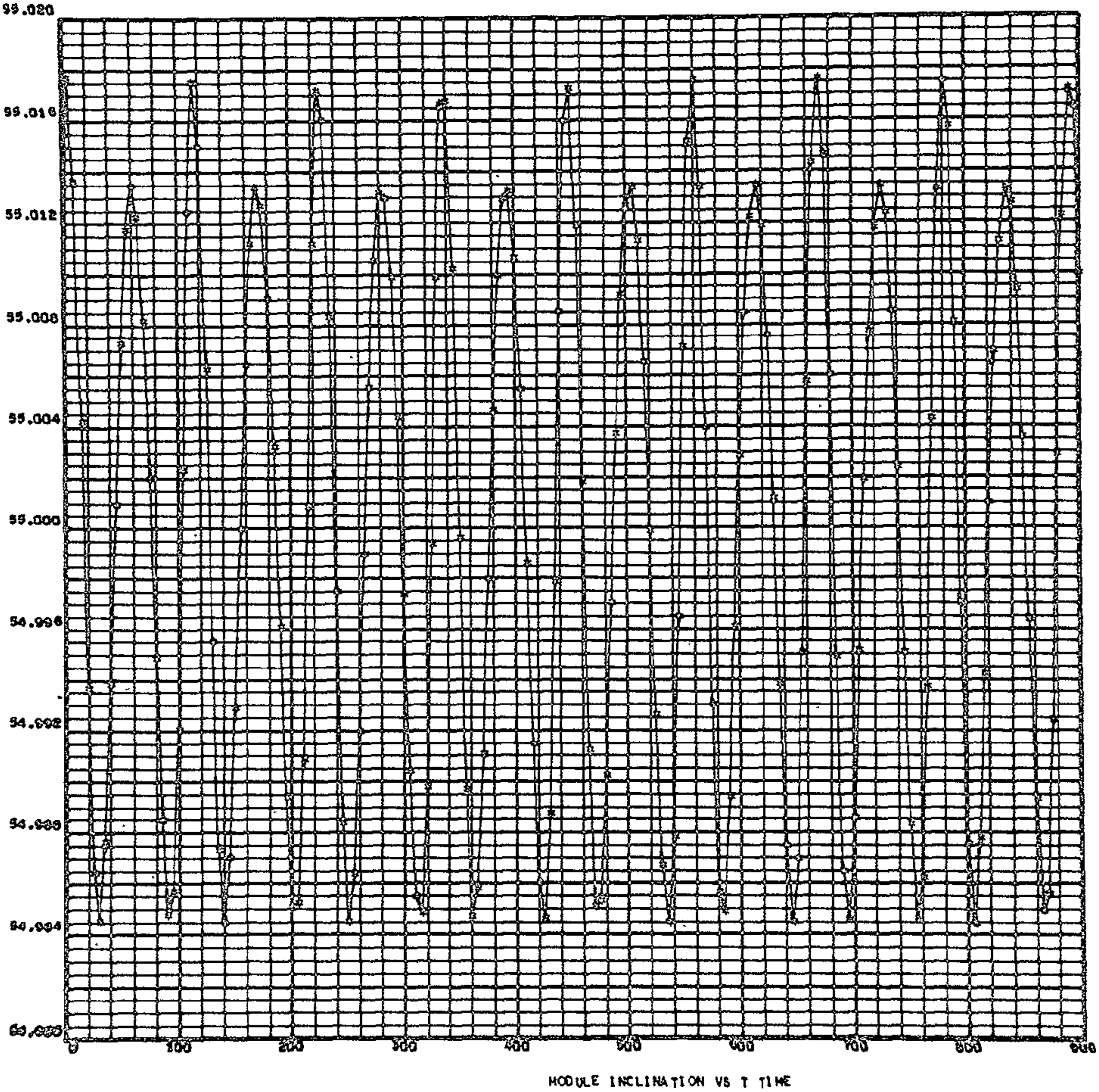


Fig. 41 – Modules Inclination (deg) vs. Time (min)
(Koelle's Equations)

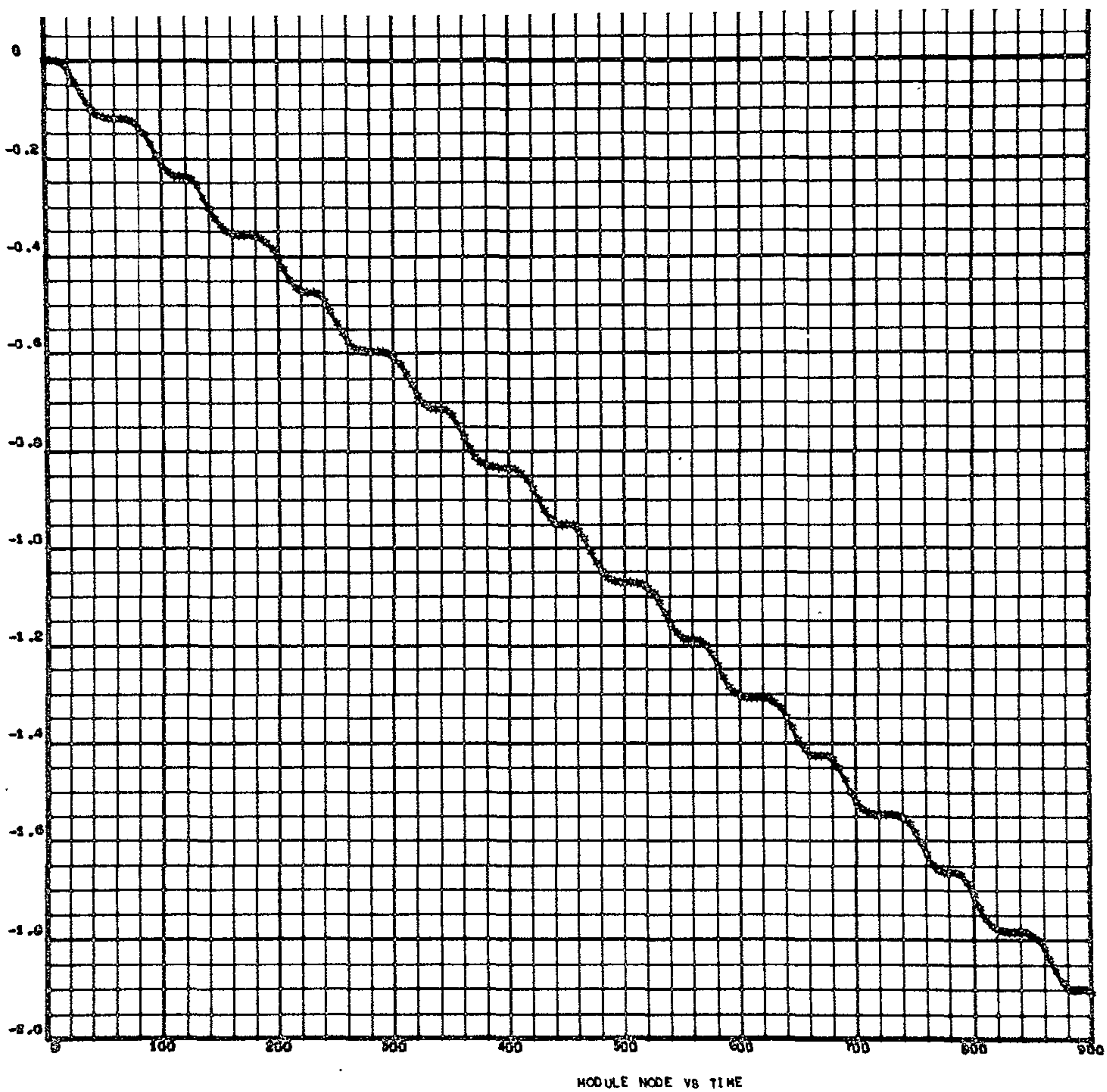


Fig. 42 – Modules Ascending Node (deg) vs. Time (min) (Koelle's Equations)

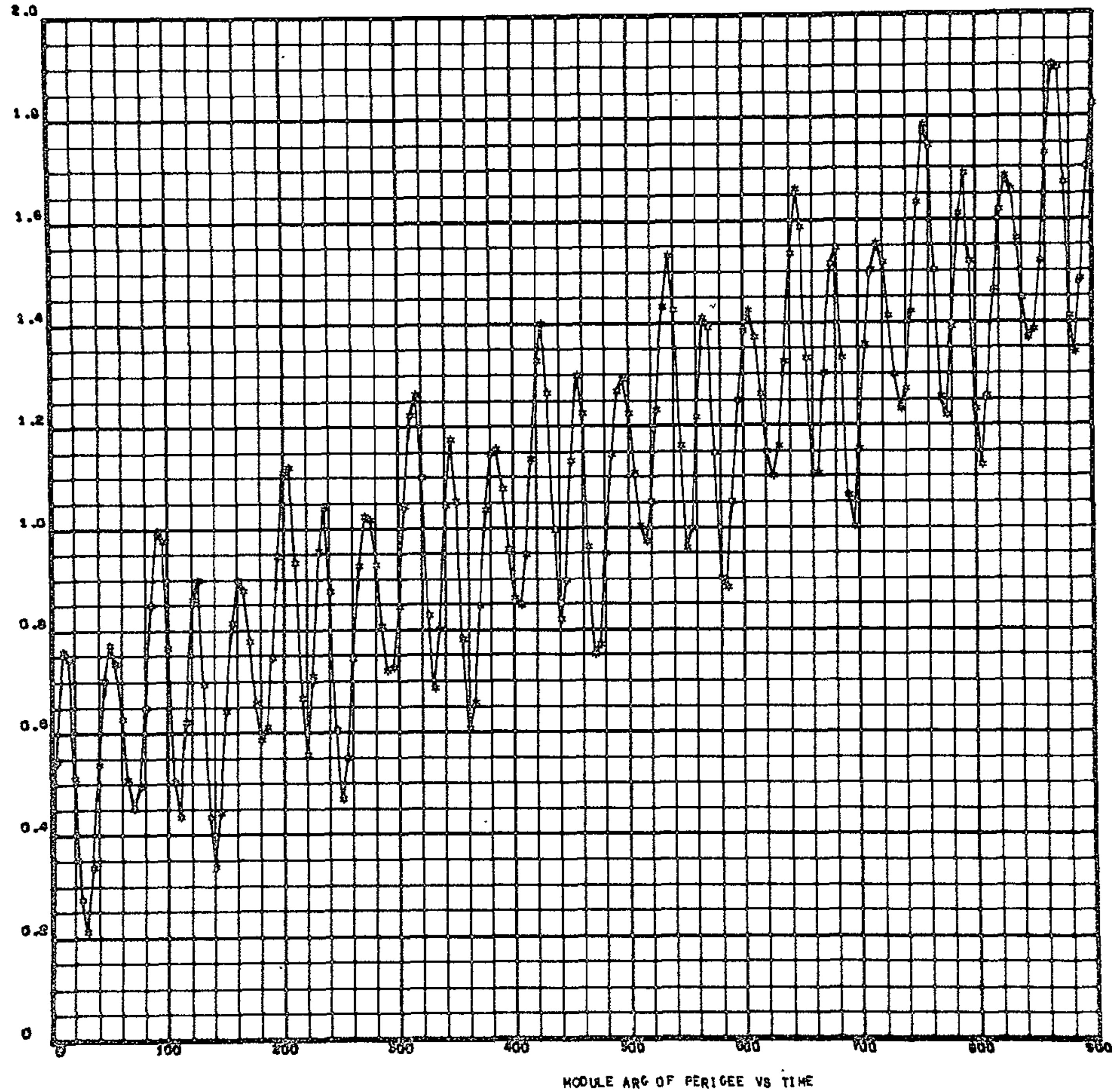


Fig. 43 – Modules Argument of Perigee (deg) vs. Time (min)
(Koelle's Equations)

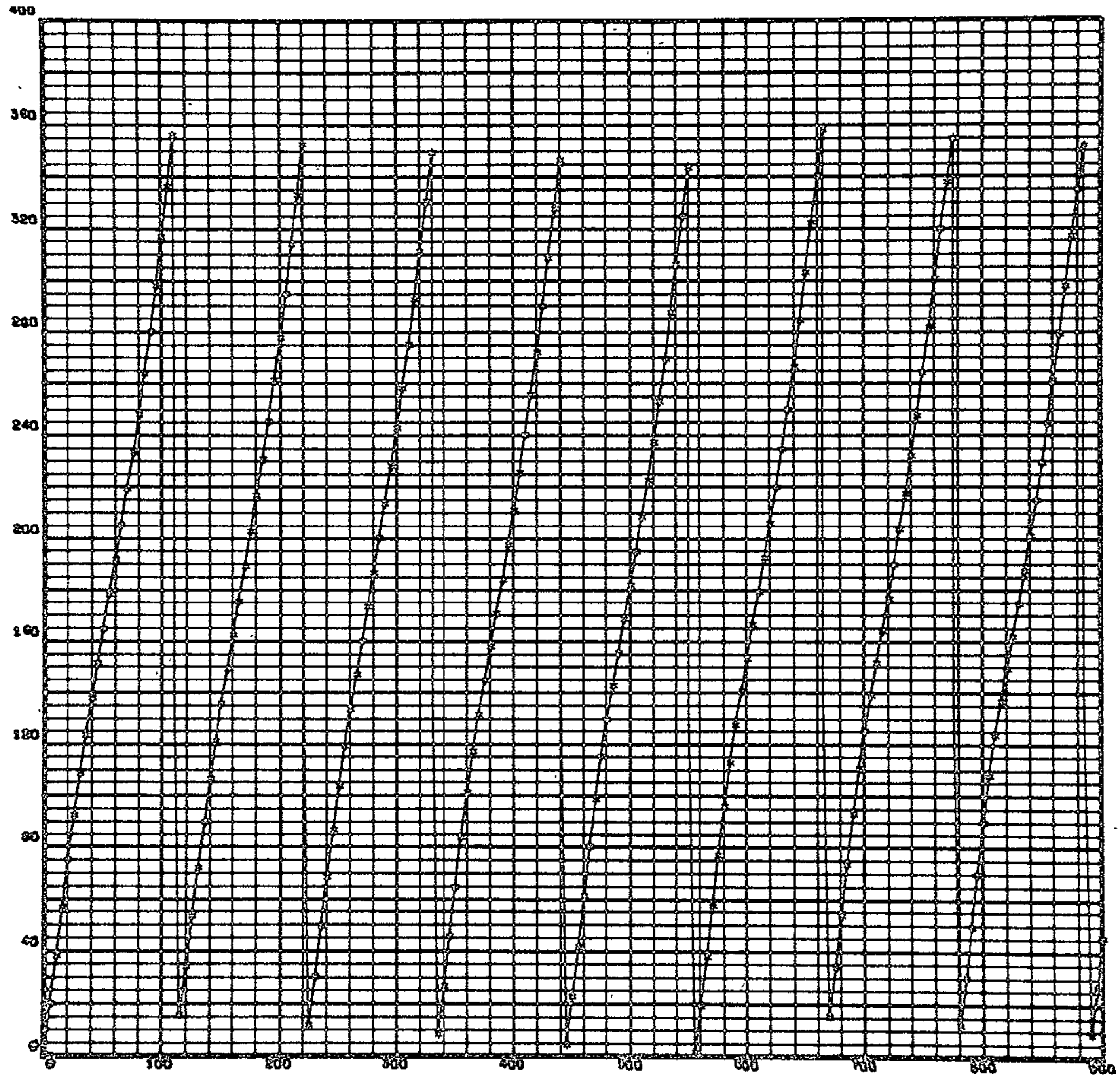


Fig. 44 — Modules True Anomaly (deg) vs. Time (min)
(Koelle's Equations)

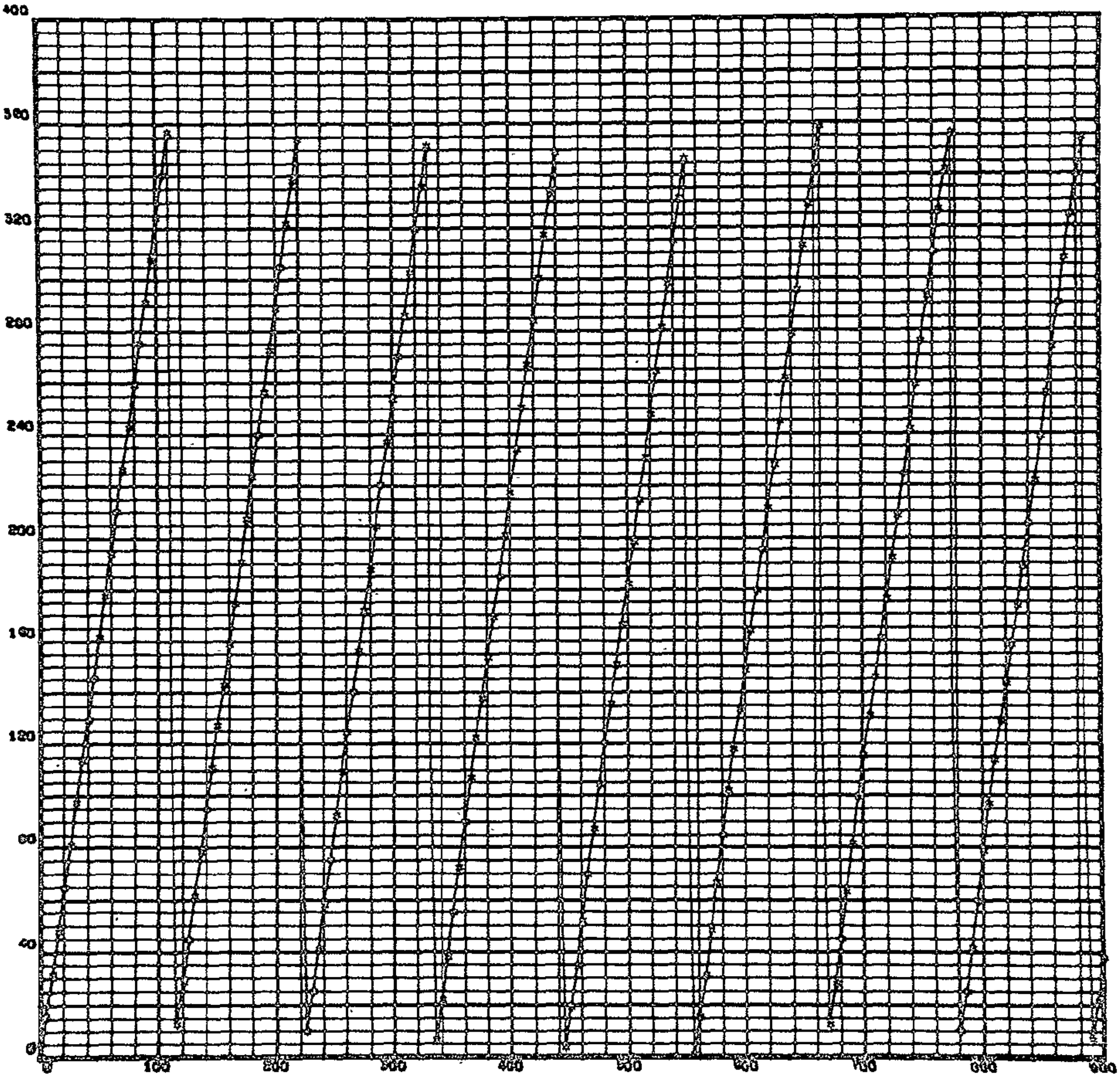


Fig. 45 – Modules Mean Anomaly (deg) vs. Time (min)
(Koelle's Equations)

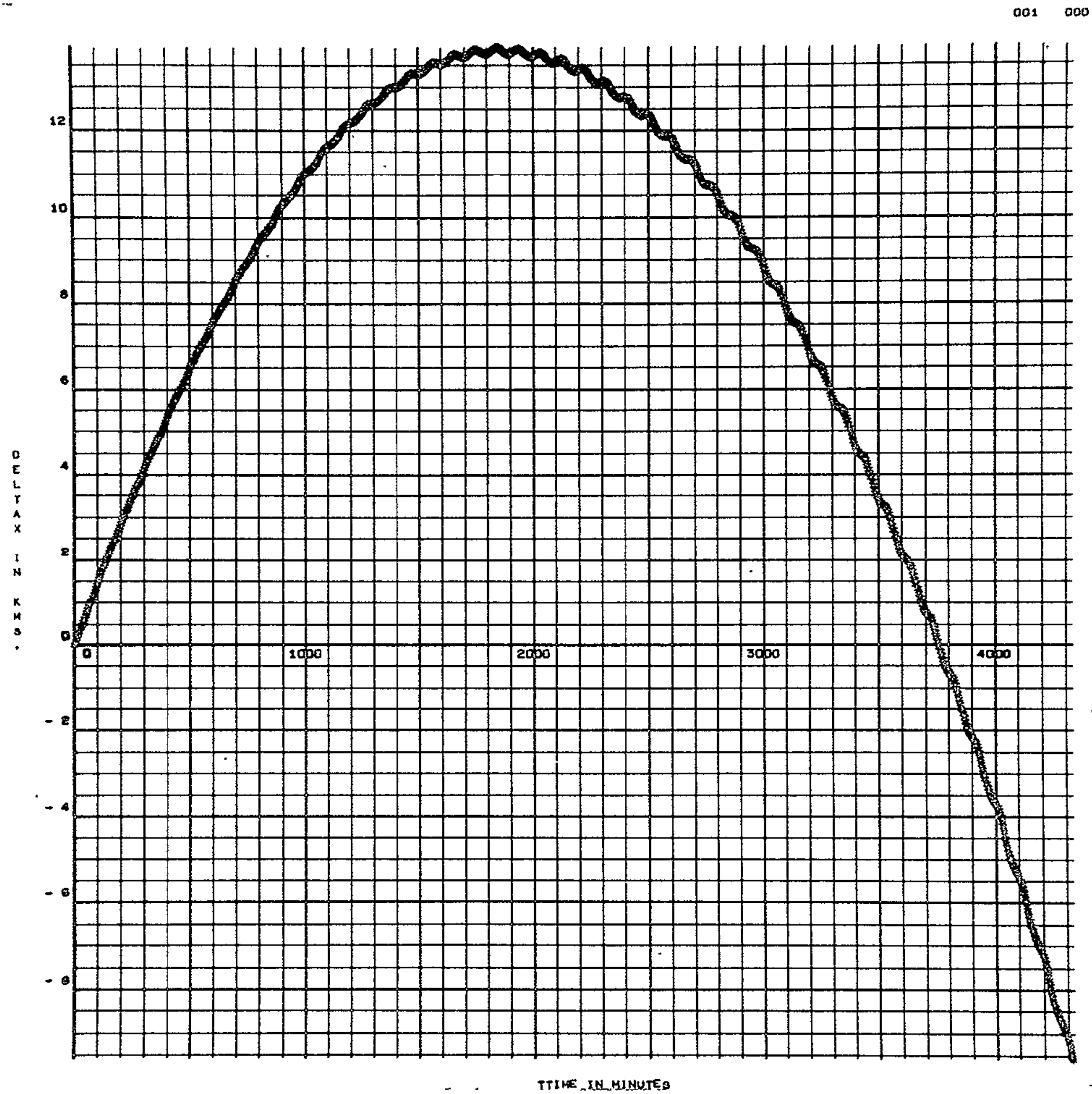


Fig. 46 – Modules X-Relative Position (km) vs. Time (min)-Loop Case

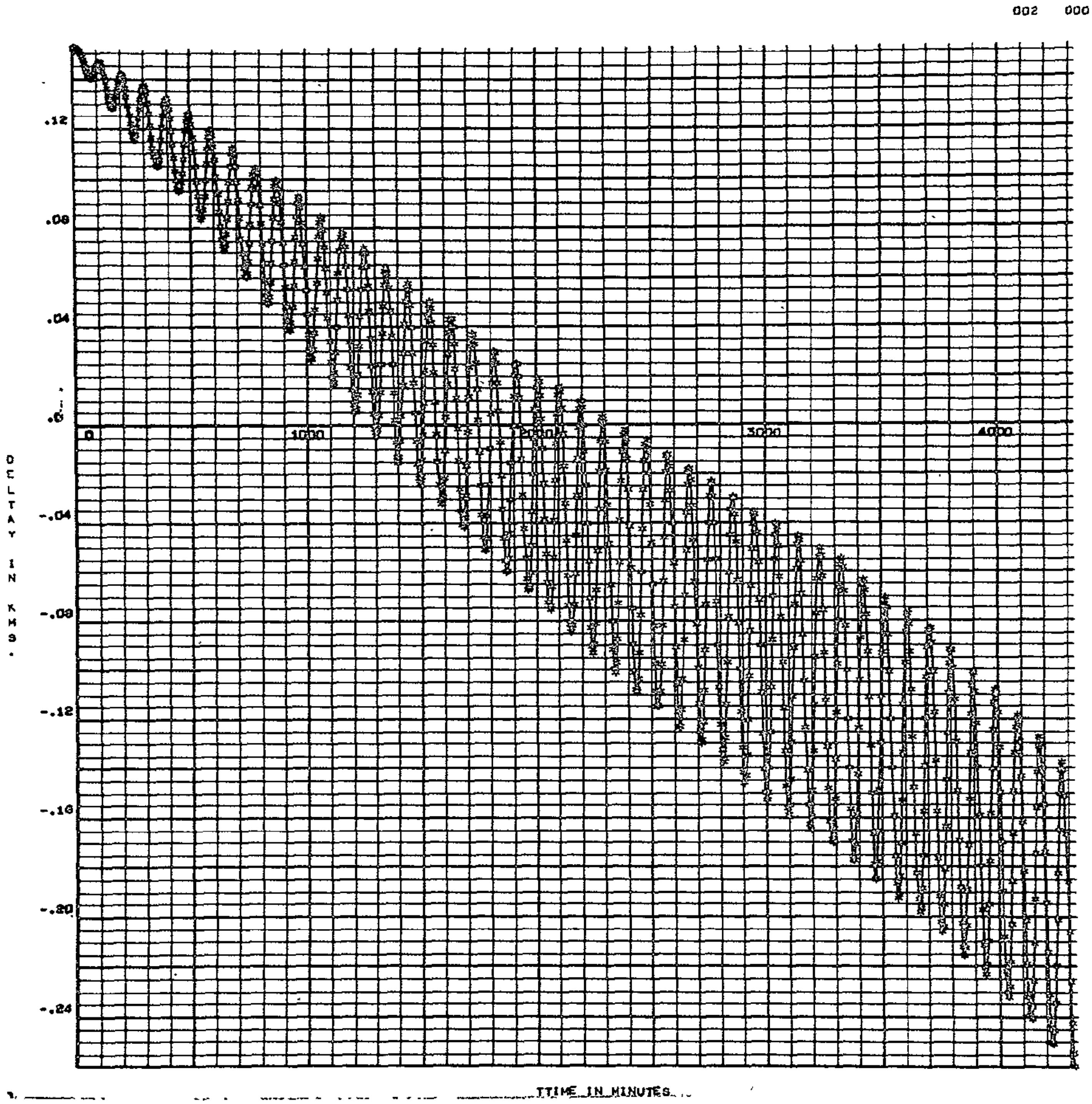


Fig. 47 – Modules Y-Relative Position (km) vs. Time (min) Loop Case

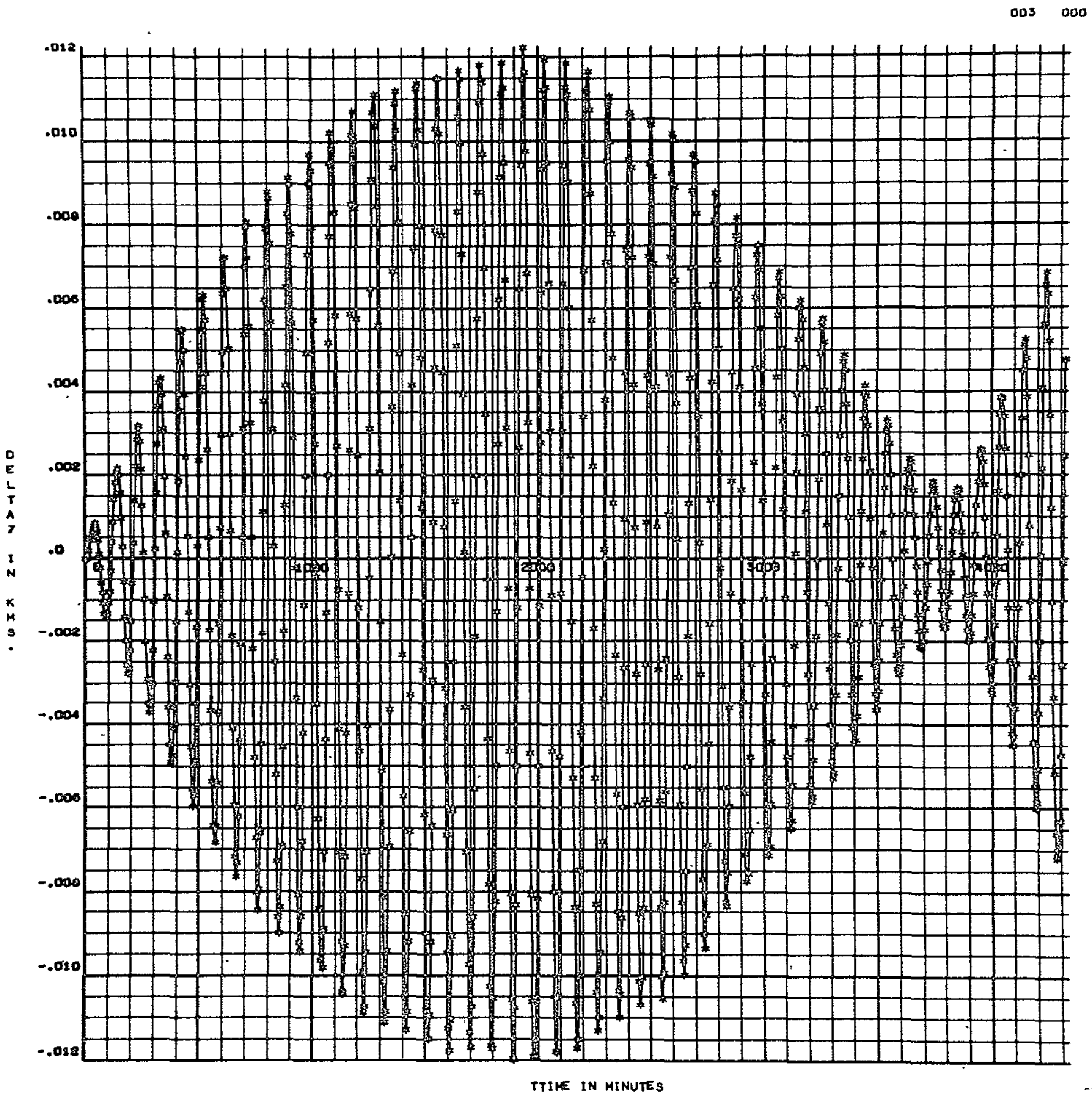


Fig. 48 – Modules Z-Relative Position (km) vs. Time (min)-loop Case

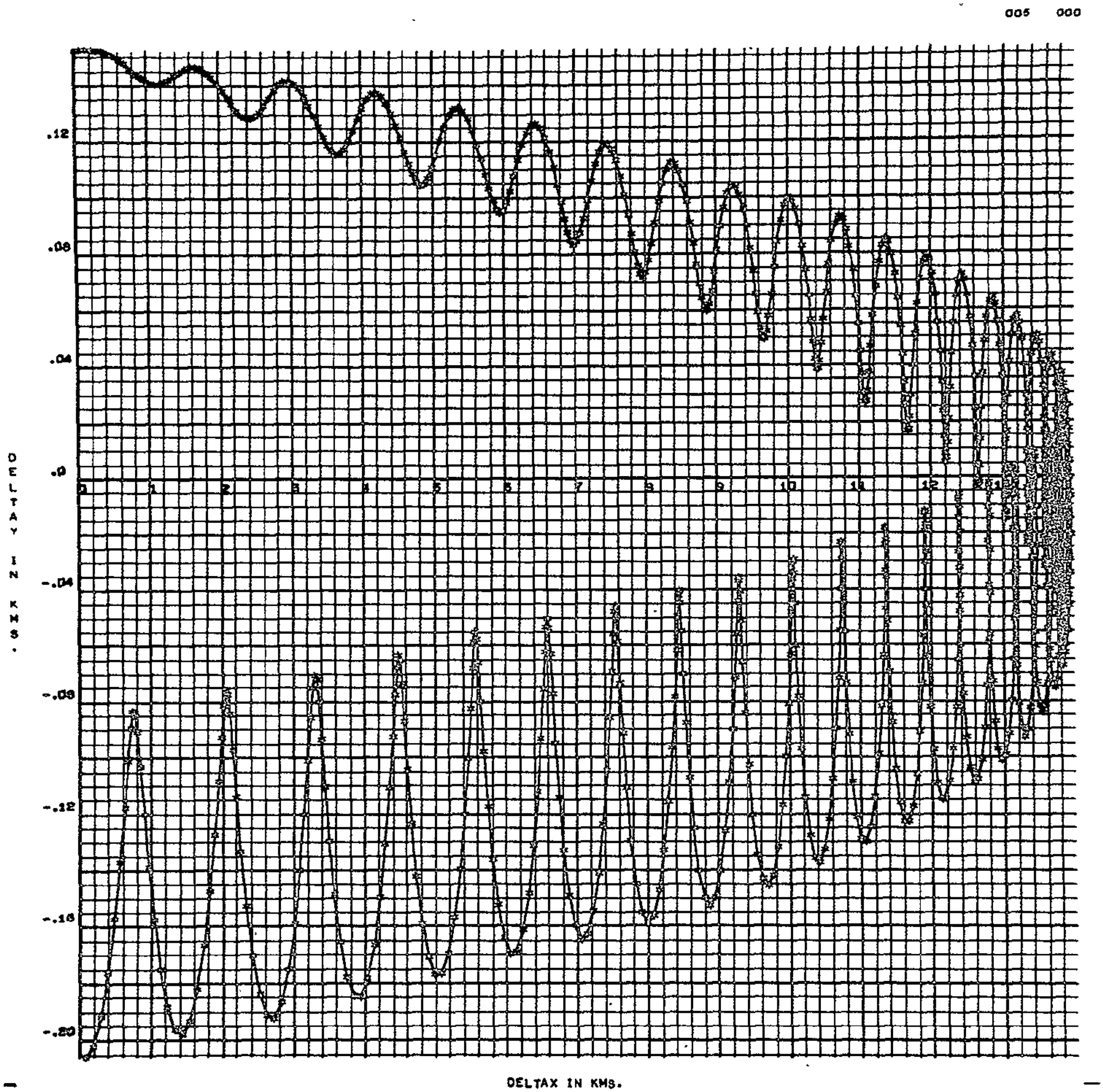


Fig. 49 – Modules Y-Relative Position (km) vs. Modules X-Relative Position (km) – Motion in Stations Plane

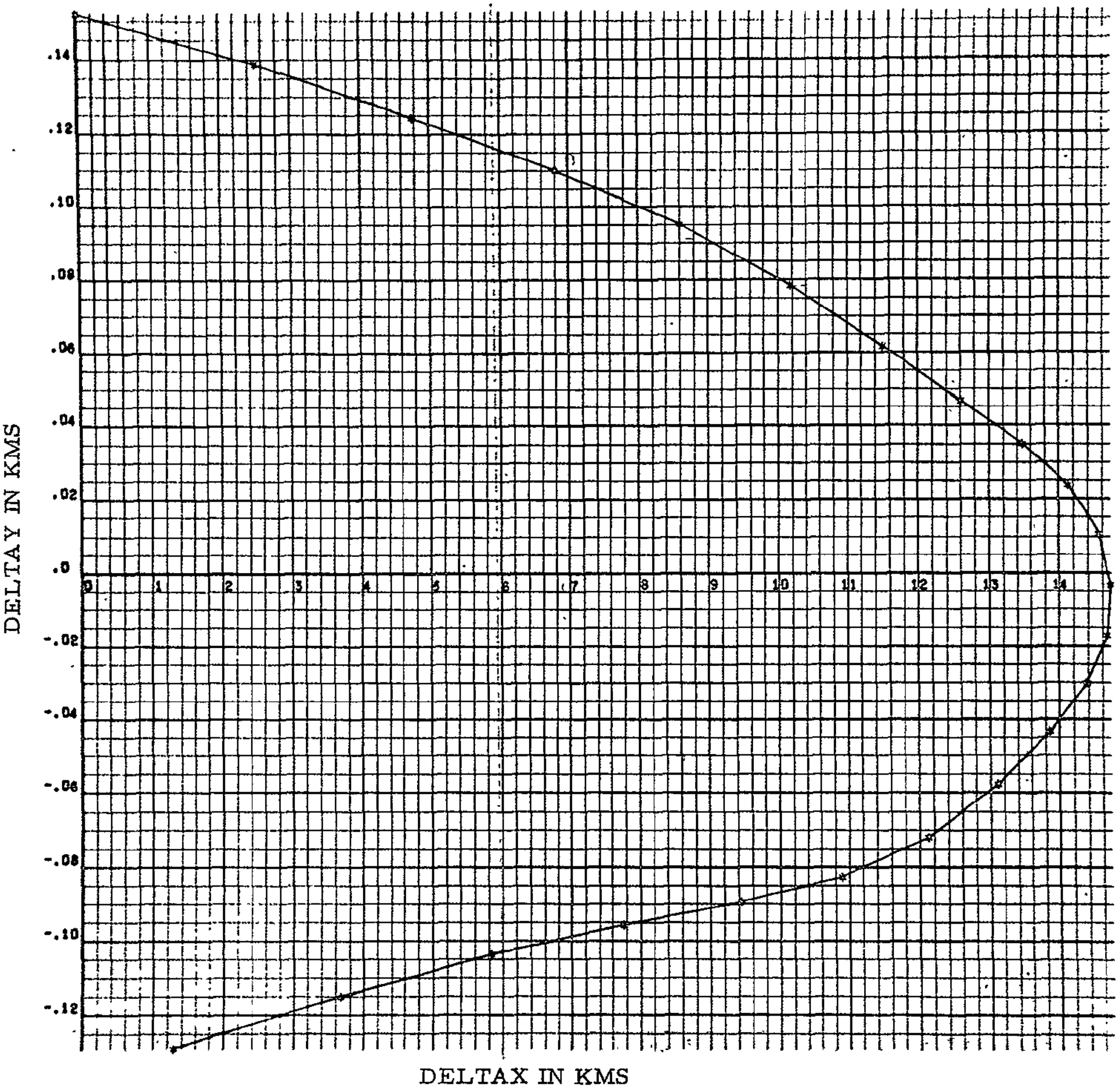


Fig. 50. - Y-Component vs X-Component (Earth Orbital Lifetime Deck)

# Appendix A

# COWELL METHOD

## Appendix A

The equations of motion to be employed are

$$\ddot{X} = \ddot{X}_{CBE} + \ddot{X}_{2HE} + \ddot{X}_{3HE} + \ddot{X}_{4HE} + \ddot{X}_{DRAG}$$

$$\ddot{Y} = \ddot{Y}_{CBE} + \ddot{Y}_{2HE} + \ddot{Y}_{3HE} + \ddot{Y}_{4HE} + \ddot{Y}_{DRAG}$$

$$\ddot{Z} = \ddot{Z}_{CBE} + \ddot{Z}_{2HE} + \ddot{Z}_{3HE} + \ddot{Z}_{4HE} + \ddot{Z}_{DRAG}$$

The central body (earth) terms are

$$\ddot{X}_{CBE} = -\frac{\mu X}{R^3}$$

$$\ddot{Y}_{CBE} = -\frac{\mu Y}{R^3}$$

$$\ddot{Z}_{CBE} = -\frac{\mu Z}{R^3}$$

The terms due to the second harmonic of the earth's potential are

$$\ddot{X}_{2HE} = \frac{-\mu J X A_E^2}{R^5}\left(1 - \frac{5Z^2}{R^2}\right)$$

$$\ddot{Y}_{2HE} = \frac{-\mu J Y A_E^2}{R^5}\left(1 - \frac{5Z^2}{R^2}\right)$$

$$\ddot{Z}_{2HE} = \frac{-\mu J Z A_E^2}{R^5}\left(3 - \frac{5Z^2}{R^2}\right)$$

The terms due to the third harmonic of the earth's potential are

$$\ddot{X}_{3HE} = \frac{\mu HXZ\, A_E^3}{R^7}\left(3 - \frac{7Z^2}{R^2}\right)$$

$$\ddot{Y}_{3HE} = \frac{\mu HYZ\, A_E^3}{R^7}\left(3 - \frac{7Z^2}{R^2}\right)$$

$$\ddot{Z}_{3HE} = \frac{-3\,\mu H\, A_E^3}{5R^5}\left(1 - \frac{10Z^2}{R^2} + \frac{35Z^4}{3R^4}\right)$$

The terms due to the fourth harmonic of the earth's potential are

$$\ddot{X}_{4HE} = \frac{-\,\mu DX\, A_E^4}{R^7}\left(\frac{3}{7} - 6\,\frac{Z^2}{R^2} + 9\,\frac{Z^4}{R^4}\right)$$

$$\ddot{Y}_{4HE} = \frac{-\,\mu DY\, A_E^4}{R^7}\left(\frac{3}{7} - 6\,\frac{Z^2}{R^2} + 9\,\frac{Z^4}{R^4}\right)$$

$$\ddot{Z}_{4HE} = \frac{-\,\mu DZ\, A_E^4}{R^7}\left(\frac{15}{7} - 10\,\frac{Z^2}{R^2} + 9\,\frac{Z^4}{R^4}\right)$$

The drag perturbation terms are

$$\ddot{X}_{DRAG} = -\frac{C_D A}{2m}\,\rho V_e\,(\dot{X} + \omega Y)\,10^3$$

$$\ddot{Y}_{DRAG} = -\frac{C_D A}{2m}\,\rho V_e\,(\dot{Y} - \omega X)\,10^3$$

$$\ddot{Z}_{DRAG} = -\frac{C_D A}{2m}\,\rho V_e\,\dot{Z}\,10^3$$

where

$\omega$ = rotational velocity of earth's atmosphere (rad/sec)

$A_E$ = mean radius of earth ellipsoid (km)

$\mu$ = earth gravitational constant ($km^3/sec^2$)

J, H, D = second, third, fourth constants of the earth potential function

where

$J = 3/2\ J_2 = +\ 1624 \times 10^{-6}$

$H = -5/2\ J_3 = +0.575 \times 10^{-5}$

$D = -15/4\ J_4 = +\ 0.795 \times 10^{-5}$

$\rho$ = atmospheric density ($kg/m^3$)

$V_e$ = inertial velocity (km/sec)

$C_D$ = dimensionless drag coefficient

A = area (frontal) of vehicle ($m^2$)

m = mass of vehicle (kg)

The equations of motion ($\ddot{X}$, $\ddot{Y}$, $\ddot{Z}$) are integrated numerically using fourth-order Runge-Kutta integration to establish the geocentric space-fixed velocities ($\dot{X}$, $\dot{Y}$, $\dot{Z}$) and position (X, Y, Z). The units on all acceleration terms are $km/sec^2$.

# Appendix B

# KOELLE'S VARIATION OF PARAMETERS GENERAL PERTURBATION TECHNIQUE

## Appendix B

Given initial values of semi-major axis $a_o$, eccentricity $e_o$, inclination $i_o$, argument of perigee $\omega_o$, ascending node $\Omega_o$, and mean anomaly $M_o$, then at any given time, these orbital elements are given by

$$a = a_o + \frac{\frac{3}{2} J_2 a_e^2}{a(1-e^2)^3} \left\{ (2 - 3\sin^2 i) \left[ e\left(1 + \frac{1}{4}e^2\right)\cos v + \frac{1}{2}e^2 \cos 2v + \frac{1}{12}e^3 \cos 3v \right] \right.$$

$$+ \sin^2 i \left[ \frac{1}{8}e^3 \cos(-v + 2\omega) + \frac{3}{4}e^2 \cos 2\omega + \frac{3}{2}e\left(1 + \frac{1}{4}e^2\right)\cos(v + 2\omega) \right.$$

$$+ \left(1 + \frac{3}{2}e^2\right) \cos(2v + 2\omega) + \frac{3}{2}e\left(1 + \frac{1}{4}e^2\right)\cos(3v + 2\omega)$$

$$\left.\left. + \frac{3}{4}e^2 \cos(4v + 2\omega) + \frac{1}{8}e^3 \cos(5v + 2\omega) \right] \right\}$$

$$e = e_o + \frac{3}{4} J_2 \left(\frac{a_e}{P}\right)^2 \left\{ (2 - 3\sin^2 i) \left[ \left(1 + \frac{1}{4}e^2\right)\cos v + \frac{1}{2}e \cos 2v + \frac{1}{12}e^2 \cos 3v \right] \right.$$

$$+ \sin^2 i \left[ \frac{1}{8}e^2 \cos(-v + 2\omega) + \frac{3}{4}e \cos 2\omega + \frac{1}{2}\left(1 + \frac{11}{4}e^2\right)\cos(v + 2\omega) \right.$$

$$+ \frac{5}{2}e \cos(2v + 2\omega) + \frac{1}{6}\left(7 + \frac{17}{4}e^2\right)\cos(3v + 2\omega) + \frac{3}{4}e \cos(4v + 2\omega)$$

$$\left.\left. + \frac{1}{8}e^2 \cos(5v + 2\omega) \right] \right\} + \frac{J_2 a_e^2}{16} \frac{e \sin i}{a P(4 - 5\sin^2 i)} \left[ (14 - 15\sin^2 i) \right.$$

$$\left. + 5\frac{J_4}{J_2^2}(6 - 7\sin^2 i) \right] \cos 2\omega - \frac{1}{2}\frac{J_3}{J_2}\frac{a_e}{a} \sin i \sin\omega$$

(continued)

$$+ \frac{5}{32} \frac{J_5 a_e^3}{J_2} \frac{\sin i}{a p^2 (4 - 5\sin^2 i)} \left\{ (4 + 3e^2) \left[ 8 - 7\sin^2 i (4 - 3\sin^2 i) \right] \sin\omega \right.$$

$$\left. + \frac{7}{6} e^2 \sin^2 i (8 - 9\sin^2 i) \sin 3\omega \right\}$$

$$i = i_o + \frac{1}{8} J_2 \left(\frac{a_e}{P}\right)^2 \sin 2i \left[ 3e\cos(v + 2\omega) + 3\cos(2v + 2\omega) + e\cos(3v + 2\omega) \right]$$

$$- \frac{1}{32} J_2 \left(\frac{a_e}{P}\right)^2 \frac{e^2 \sin 2i}{4 - 5\sin^2 i} \left[ (14 - 15\sin 2i) + 5 \frac{J_4}{J_2^2} (6 - 7\sin^2 i) \right] \cos 2\omega$$

$$+ \frac{1}{2} \frac{J_3}{J_2} \frac{a_e}{P} e\cos i \sin\omega - \frac{5}{32} \frac{J_5}{J_2} \left(\frac{a_e}{P}\right)^3 \frac{e\cos i}{4 - 5\sin^2 i} \left\{ (4 + 3e^2) \right.$$

$$\left. \times \left[ 8 - 7\sin^2 i (4 - 3\sin^2 i) \right] \sin\omega + \frac{7}{16} e^2 \sin^2 i (8 - 9\sin^2 i) \sin 3\omega \right\}$$

$$\omega = \omega_o + \frac{3}{4} J_2 \left(\frac{a_e}{P}\right)^2 (4 - 5\sin^2 i) nt \left\{ 1 + \frac{1}{32} J_2 \left(\frac{a_e}{P}\right)^2 \left[ 24(4 + e^2) \right. \right.$$

$$\left. \left. - \sin^2 i (86 - e^2) \right] \right\} - \frac{15}{16} J_2^2 \left(\frac{a_e}{P}\right)^4 e^2 \cos^4 i\, nt$$

$$- \frac{15}{128} J_4 \left(\frac{a_e}{P}\right)^4 nt \left[ 64 - 248\sin^2 i + 196 \sin^4 i \right.$$

$$\left. + e^2 (72 - 252\sin^2 i + 189\sin^4 i) \right] + \frac{3}{2} J_2 \left(\frac{a_e}{P}\right)^2 \frac{1}{e} \left\{ \frac{1}{2} e(4 - 5\sin^2 i)(v - M + e\sin v) \right.$$

$$+ \left(1 - \frac{1}{4} e^2\right) \sin v + \frac{1}{2} e\sin 2v + \frac{1}{12} e^2 \sin 3v - \frac{1}{2} e^2 \sin(v + 2\omega)$$

(continued)

$$- \frac{1}{2} e \sin(2v + 2\omega) - \frac{1}{6} e^2 \sin(3v + 2\omega) + \sin^2 i \left[ - \frac{3}{2}\left(1 - \frac{1}{4} e^2\right) \sin v \right.$$

$$- \frac{3}{4} e \sin 2v - \frac{1}{8} e^2 \sin 3v - \frac{1}{16} e^2 \sin(-v + 2\omega) + \frac{1}{4}\left(-1 + \frac{15}{4} e^2\right) \sin(v + 2\omega)$$

$$+ \frac{5}{4} e \sin(2v + 2\omega) + \frac{1}{12}\left(7 + \frac{19}{4} e^2\right) \sin(3v + 2\omega) + \frac{3}{8} e \sin(4v + 2\omega)$$

$$\left. \left. + \frac{1}{16} e^2 \sin(5v + 2\omega) \right] \right\} + \frac{3}{8} J_2 \left(\frac{a_e}{P}\right)^2 \frac{1}{4 - 5 \sin^2 i} \left\{ \frac{7}{3} e^2 - \frac{1}{6} (50 + 79e^2) \sin^2 i \right.$$

$$+ \frac{5}{4} (8 + 9e^2) \sin^4 i + \frac{e^2 \sin^2 i (13 - 15 \sin^2 i)(14 - 15 \sin^2 i)}{6(4 - 5 \sin^2 i)}$$

$$- \frac{5}{6} \frac{J_4}{J_2^2} \left[ \sin^2 i (6 - 7 \sin^2 i) - \frac{1}{2} e^2 (12 - 70 \sin^2 i - 63 \sin^4 i) \right.$$

$$\left. \left. - e^2 \sin^2 i \frac{(13 - 15 \sin^2 i)(6 - 7 \sin^2 i)}{4 - 5 \sin^2 i} \right] \right\} \sin 2\omega$$

$$+ \frac{1}{2} \frac{J_3}{J_2} \frac{a_e}{P} \left( \frac{e \cos^2 i}{\sin i} - \frac{1}{e} \sin i \right) \cos\omega$$

$$- \frac{5}{32} \frac{J_5}{J_2} \left(\frac{a_e}{P}\right)^3 \frac{1}{4 - 5 \sin^2 i} \left\{ \left[ \frac{e^2 - \sin^2 i}{e \sin i} (4 + 3e^2) \right. \right.$$

$$\left. - e(26 + 9e^2) \sin i \right] \left[ 8 - 7 \sin^2 i (4 - 3 \sin^2 i) \right]$$

$$\left. - \frac{6e(4 + 3e^2) \cos^2 i \sin i}{4 - 5 \sin^2 i} \left[ 24 - 7 \sin^2 i (8 - 5 \sin^2 i) \right] \right\} \cos\omega$$

(continued)

$$- \frac{35}{576} \frac{J_5}{J_2} \left(\frac{a_e}{P}\right)^3 \frac{1}{4 - 5\sin^2 i} \left\{ \left[ e^3 \sin i - 3e(1 + e^2) \sin^3 i \right] (8 - 9 \sin^2 i) \right.$$

$$\left. + 2e^3 \cos^2 i \sin i \left[ (8 - 9 \sin^2 i) + \frac{4 \sin^2 i}{4 - 5 \sin^2 i} \right] \right\} \cos 3\omega$$

$$\Omega = \Omega_o - \frac{3}{2} J_2 \left(\frac{a_e}{P}\right)^2 \cos i \, nt \left\{ 1 - \frac{5}{16} \frac{J_4}{J_2} \left(\frac{a_e}{P}\right)^2 (2 + 3e^2)(4 - 7 \sin^2 i) \right.$$

$$\left. + \frac{1}{16} J_2 \left(\frac{a_e}{P}\right)^2 \left[ 4(9 + e^2) - 5 \sin^2 i (8 - e^2) \right] \right\}$$

$$- \frac{1}{4} J_2 \left(\frac{a_e}{P}\right)^2 \cos i \left[ 6(v - M + e \sin v) - 3e \sin(v + 2\omega) - 3 \sin(2v + 2\omega) \right.$$

$$\left. - e \sin(3v + 2\omega) \right] - \frac{1}{16} J_2 \left(\frac{a_e}{P}\right)^2 \frac{e^2 \cos i}{4 - 5 \sin^2 i} \left\{ 2(7 - 15 \sin^2 i) \right.$$

$$\left. + 5 \sin^2 i \frac{14 - 15 \sin^2 i}{4 - 5 \sin^2 i} + 5 \frac{J_4}{J_2} \left[ 2(3 - 7 \sin^2 i) + 5 \sin^2 i \frac{6 - 7 \sin^2 i}{4 - 5 \sin^2 i} \right] \right\} \sin 2\omega$$

$$- \frac{1}{2} \frac{J_3}{J_2} \frac{a_e}{P} e \cot i \cos\omega + \frac{5}{32} \frac{J_5}{J_2} \left(\frac{a_e}{P}\right)^3 \frac{e \cot i}{4 - 5 \sin^2 i} \left\{ \frac{10(4 + 3e^2) \sin^2 i}{4 - 5 \sin^2 i} \right.$$

$$\times \left[ 8 - 7 \sin^2 i (4 - 3 \sin^2 i) \right] \cos\omega + (4 + 3e^2)$$

$$\times \left[ 8 - 21 \sin^2 i (4 - 5 \sin^2 i) \right] \cos\omega + \frac{7}{18} e^2 \sin^2 i$$

$$\left. \times \left[ 3(8 - 15 \sin^2 i) + 10 \sin^2 i \frac{8 - 9 \sin^2 i}{4 - 5 \sin^2 i} \right] \cos 3\omega \right\}$$

$$M = M_o + nt\left[1 + \frac{3}{2}J_2\left(\frac{a_e}{P}\right)^2\left(1 - \frac{3}{2}\sin^2 i\right)\sqrt{1-e^2}\left\{1 + \frac{1}{8}J_2\left(\frac{a_e}{P}\right)^2\left[10 + 5e^2\right.\right.\right.$$

$$\left.\left. + 8\sqrt{1-e^2} - \frac{1}{4}\sin^2 i\left(\frac{130}{3} - \frac{25}{3}e^2 + 48\sqrt{1-e^2}\right)\right]\right\}$$

$$+ \frac{5}{64}J_2^2\left(\frac{a_e}{P}\right)^4\sqrt{1-e^2}\,(2-e^2)\sin^2 i$$

$$\left. - \frac{45}{128}J_4\left(\frac{a_e}{P}\right)^4 e^2\sqrt{1-e^2}\,(8 - 40\sin^2 i + 35\sin^4 i)\right]$$

$$- \frac{3}{2}J_2\left(\frac{a_e}{P}\right)^2\sqrt{1-e^2}\left\{\left(1 - \frac{3}{2}\sin^2 i\right)\left[\left(\frac{1}{e} - \frac{e}{4}\right)\sin v + \frac{1}{2}\sin 2v\right.\right.$$

$$\left. + \frac{1}{12}e\sin^3 v\right] - \sin^2 i\left[\frac{1}{4}\left(\frac{1}{e} + \frac{5}{4}e\right)\sin(v+2\omega) - \frac{1}{16}e\sin(v-2\omega)\right.$$

$$\left.\left. - \frac{7}{12}\left(\frac{1}{e} - \frac{e}{28}\right)\sin(3v+2\omega) - \frac{3}{8}\sin(4v+2\omega) - \frac{1}{16}e\sin(5v+2\omega)\right]\right\}$$

$$+ \frac{1}{16}J_2\left(\frac{a_e}{P}\right)^2(1-e^2)^{3/2}\frac{\sin^2 i}{4 - 5\sin^2 i}\left[(14 - 15\sin^2 i)\right.$$

$$\left. + 5\frac{J_4}{J_2^2}(6 - 7\sin^2 i)\right]\sin 2\omega + \frac{1}{2}\frac{J_3}{J_2}\left(\frac{a_e}{P}\right)\frac{(1-e^2)^{3/2}}{e}\sin i\cos\omega$$

$$+ \frac{5}{32}\frac{J_5}{J_2}\left(\frac{a_e}{P}\right)^3\frac{(1-e^2)^{3/2}}{e}\frac{\sin i}{(4 - 5\sin^2 i)}\left\{(4+9e^2)\left[8 - 7\sin^2 i(4 - 3\sin^2 i\right]\cos\omega\right.$$

$$\left. - \frac{7}{6}e^2\sin^2 i\,(8 - 9\sin^2 i)\cos 3\omega\right\}$$

where

$a_e$ = earth's equatorial radius (km)

$J_2, J_3, J_4, J_5$ = earth's second, third, fourth and fifth geopotential coefficients

$J_2$ = $+ 1082.28 \times 10^{-6}$

$J_3$ = $-2.3 \times 10^{-6}$

$J_4$ = $-2.12 \times 10^{-6}$

$J_5$ = $-0.2 \times 10^{-6}$

$v$ = true anomaly (deg)

$P$ = $a(1 - e^2)$

$n$ = mean motion of vehicle

$t$ = time

# Appendix C

# PROGRAM LISTING

```
•RUN,//T BUTLER,422330,BUTLERBIN202,3,300/000    . BUTLER/LOCKHEED
•FOR,IS  MAIN,MAIN
      IMPLICIT REAL*8 (A-H,O-Z)
C THIS PROGRAM DETERMINES THE RELATIVE POSITION OF AN ASTRONOMY MODULE
C WITH RESPECT TO A SPACE STATION BY UTILIZING THE COWELL INTEGRATION
C ROUTINE. THE PROGRAM IS CONSTRUCTED TO HANDLE PERTURBATIONS DUE TO
C THE OBLATE EARTH AND ATMOSPHERIC DRAG.
      COMMON/XK/NP,NDTP,DT
      DIMENSION AS(14),AM(14),SA(14),XMA(14)
      DIMENSION TBSA(14),TBMA(14)
      DIMENSION SIGMA(14),ETA(14)
      DIMENSION PLOT5(1000),PLOT6(1000),PLOT7(1000)
      DOUBLE PRECISION MEANPS,MEANPM,MEANOS,MEANOM
      DIMENSION PLOT(1000),PLOT1(1000),PLOT2(1000),PLOT3(1000)
      DIMENSION PLOTX(1000),PLOTY(1000),PLOTZ(1000)
      COMMON/DATAB/XLAB(12),YLAB1(12),YLAB2(12),YLAB3(12),YLAB4(12),
     1YLAB5(12),YLAB6(12),YLAB7(12),YLAB8(12),YLAB9(12),YLAB10(12),
     2YLAB11(12),YLAB12(12),YLAB13(12),YLAB14(12),YLAB15(12),YLAB16(12),
     3YLAB17(12),YLAB18(12),YLAB19(12),YLAB20(12),YLAB21(12)
      REAL
     $ PLOT,PLOT1,PLOT2,PLOT3,PLOTX,PLOTY,PLOTZ,XLAB,YLAB1,
     $YLAB2,YLAB3,YLAB4
      REAL PLOT5,PLOT6,PLOT7,YLAB5,YLAB6,YLAB7
      DIMENSION PLOT8(1000),PLOT9(1000),PLOT10(1000),PLOT11(1000),
     1PLOT12(1000),PLOT13(1000),PLOT14(1000),PLOT15(1000),PLOT16(1000),
     2PLOT17(1000),PLOT18(1000),PLOT19(1000),PLOT20(1000),PLOT21(1000)
      REAL PLOT8,PLOT9,PLOT10,PLOT11,PLOT12,PLOT13,PLOT14,PLOT15,PLOT16,
     1PLOT17,PLOT18,PLOT19,PLOT20,PLOT21,YLAB8,YLAB9,YLAB10,YLAB11,
     2YLAB12,YLAB13,YLAB14,YLAB15,YLAB16,YLAB17,YLAB18,YLAB19,YLAB20,
     3YLAB21
      CALL IDENT(9)
      PI2=1.570795
      RPD = 0.01745329
C READ INITIAL ORBITAL ELEMENTS FOR STATION AND MODULE
      READ(5,20) AIPS,EIPS,FINCPS,CAPWS,SMAWS,PNUIS,AIPM,EIPM,FINCPM,
     1CAPWM,SMAWM,PNUIM
   20 FORMAT(6D12.0)
C PRINT IDENTIFICATION OF OUTPUT ORBITAL PARAMETERS OF STATION AND
C MODULE AND THE COORDINATES OF THE MODULE WITH RESPECT TO THE STATION
      WRITE(6,50)
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
  50 FORMAT(1H1, 40X, 36H*****DESCRIPTION OF DATA OUTPUT*****/
    $ 28H0STATION TRUE ANOMALY (DEGS) /
    $ 30H STATION SEMI-MAJOR AXIS (KMS), 7X, 20HSTATION ECCENTRICITY,
    $ 9X, 65HSTATION INCLINATION (DEGS)  STATION LONG OF ASCENDING NODE
    $ (DEGS) /1X, 62HSTATION ARGUMENT OF PERIGEE (DEGS)  STATION MEAN A
    $NOMALY(DEGS) /1X, 19HSTATION TIME (MINS), 16X, 19HSTATION TIME (DA
    $YS) )
C
      WRITE (6,11)
  11 FORMAT(1H0, 26HMODULE TRUE ANOMALY (DEGS) /
    $ 29H MODULE SEMI-MAJOR AXIS (KMS), 8X, 19HMODULE ECCENTRICITY,10X,
    $ 65H MODULE INCLINATION (DEGS)   MODULE LONG OF ASCENDING NODE (DE
    $GS) /1X, 62HMODULE ARGUMENT OF PERIGEE (DEGS)   MODULE MEAN ANOMAL
    $Y (DEGS) ' )
C
      WRITE (6,12)
  12 FORMAT(1H0, 91HDELTAX OF MODULE TO STATION (KMS)   DELTAY OF MOD T
    $O STAT (KM)   DELTAZ OF MOD TO STAT (KM) /1X, 23HDEVIATION OF TWO-
    $BODY-X, 13X, 23HDEVIATION OF TWO-BODY-Y, 6X, 23HDEVIATION OF TWO-B
    $ODY-Z )
C
C     READ BALLISTIC COEFFICIENT OF STATION AND MODULE
      READ(5,778)CDAS,CDAM
 778 FORMAT(2D12.8)
C     READ INTEGRATION STEP SIZE(SEC),CUTOFF TIME(HR),INTEGRATION STEPS
C     PER PRINT INTERVAL
      READ(5,37)DT,TCUT,NP
  37  FORMAT(2D12.8,I3)
      NDTP=NP
      TSTA=0.0
      TMOD=0.0
      T2BSTA=0.0
      T2BMOD=0.0
C     TRANSFORM ORBITAL ELEMENTS TO RECTANGULAR EARTH CENTERED COORD
      CALL TRANFM(AIPS,EIPS,FINCPS,CAPWS,PNUIS,SMAWS,XS,YS,ZS,XDS,YDS,ZD
     $S)
      AS(1)=XS
      AS(4)=YS
      AS(7)=ZS
      AS(2)=XDS
```

```
      AS(5)=YDS
      AS(8)=ZDS
      CALL TRANFM(AIPM,EIPM,FINCPM,CAPWM,PNUIM,SMAWM,XM,YM,ZM,XDM,YDM,ZD
     $M)
      AM(1)=XM
      AM(4)=YM
      AM(7)=ZM
      AM(2)=XDM
      AM(5)=YDM
      AM(8)=ZDM
      SIGMA(1)=XS
      SIGMA(4)=YS
      SIGMA(7)=ZS
      SIGMA(2)=XDS
      SIGMA(5)=YDS
      SIGMA(8)=ZDS
      ETA(1)=XM
      ETA(4)=YM
      ETA(7)=ZM
      ETA(2)=XDM
      ETA(5)=YDM
      ETA(8)=ZDM
      AOS=AIPS
      EOS=EIPS
      XIOS=FINCPS
      WOS=CAPWS
      SWOS=SMAWS
      TRUS=PNUIS
      AOM=AIPM
      EOM=EIPM
      WOM=CAPWM
      XIOM=FINCPM
      SWOM=SMAWM
      TRUM=PNUIM
      TTIME=0.
      IPT=0
  101 CALL COWELL(AS,CDAS,FINCPS,CAPWS,SMAWS,MEANPS,AIPS,EIPS,PNUIS,RS,
     *SA,TSTA)
      IF(IPT.NE.0) GO TO 9991
      NP=NDTP
```

```
      GO TO 9992
 9991 NP=0
 9992 CONTINUE
      FINCPS = FINCPS/RPD
      CAPWS = CAPWS/RPD
      SMAWS = SMAWS/RPD
      XS=SA(1)
      YS=SA(4)
      ZS=SA(7)
      XDS=SA(2)
      YDS=SA(5)
      ZDS=SA(8)
      TTIMEM=TTIME/60.
      TTIMED=TTIME/86400.
      CALL COWELL(AM,CDAM,FINCPM,CAPWM,SMAWM,MEANPM,AIPM,EIPM,PNUIM,RM,
     *XMA,TMOD)
      FINCPM = FINCPM/RPD
      CAPWM = CAPWM/RPD
      SMAWM = SMAWM/RPD
      XM=XMA(1)
      YM=XMA(4)
      ZM=XMA(7)
C     COMPUTE RELATIVE POSITIONS DELTAX,DELTAY,DELTAZ,OF MODULE TO
C     STATION UNDER PERTURBATIVE EFFECTS
      RM=SQRT(XM*XM+YM*YM+ZM*ZM)
      HX=YS*ZDS-ZS*YDS
      HY=ZS*XDS-XS*ZDS
      HZ=XS*YDS-YS*XDS
      QX=YM*HZ-ZM*HY
      QY=ZM*HX-XM*HZ
      QZ=XM*HY-YM*HX
      HDRM=HX*XM+HY*YM+HZ*ZM
      ABH=SQRT(HX*HX+HY*HY+HZ*HZ)
      ABQ=SQRT(QX*QX+QY*QY+QZ*QZ)
      COPHI=HDRM/(ABH*RM)
      SIPHI=SQRT(1.-COPHI*COPHI)
      XMAGP=RM*SIPHI
      THETA=ACOS((QX*XS+QY*YS+QZ*ZS)/(ABQ*RS))
      DELTAX=XMAGP*COS(THETA)
      DELTAX=-DELTAX
```

```
      DELTAY=XMAGP*SIN(THETA)-RS
      DELTAZ=HDRM/ABH
      DELTAR=SQRT(DELTAX*DELTAX+DELTAY*DELTAY+DELTAZ*DELTAZ)
C
      IF(IPT.NE.0) GO TO 333
      NP=NDTP
      GO TO 343
  333 NP=0
  343 CONTINUE
      CALL TOBODY(SIGMA,TBSA,RTBS,AOS,EOS,XIOS,WOS,SWOS,TRUS,MEANOS,
     1T2BSTA)
      XIOS=XIOS/RPD
      SWOS=SWOS/RPD
      WOS=WOS/RPD
      IF(IPT.NE.0) GO TO 633
      NP=NDTP
      GO TO 643
  633 NP=0
  643 CONTINUE
      CALL TOBODY(ETA,TBMA,RTBM,AOM,EOM,XIOM,WOM,SWOM,TRUM,MEANOM,T2BMOD
     2)
      SWOM=SWOM/RPD
      WOM=WOM/RPD
      XIOM=XIOM/RPD
      NP=0
      IF(IPT-1000) 999,998,998
  999 IPT = IPT + 1
  998 PLOT(,IPT)=TTIME/60.
      XTBS=TBSA(1)
      YTBS=TBSA(4)
      ZTBS=TBSA(7)
      XDTBS=TBSA(2)
      YDTBS=TBSA(5)
      ZDTBS=TBSA(8)
      XTBM=TBMA(1)
      YTBM=TBMA(4)
      ZTBM=TBMA(7)
C     COMPUTE RELATIVE POSITIONS DXTB,DYTB,DZTB,OF MODULE TO STATION
C     FOR TWO-BODY MOTION
      H1=YTBS*ZDTBS-ZTBS*YDTBS
```

```
      H2=ZTBS*XDTBS-XTBS*ZDTBS
      H3=XTBS*YDTBS-YTBS*XDTBS
      Q1=YTBM*H3-ZTBM*H2
      Q2=ZTBM*H1-XTBM*H3
      Q3=XTBM*H2-YTBM*H1
      HRM=H1*XTBM+H2*YTBM+H3*ZTBM
      ABH1=SQRT(H1*H1+H2*H2+H3*H3)
      ABQ1=SQRT(Q1*Q1+Q2*Q2+Q3*Q3)
      COCHI=HRM/(ABH1*RTBM)
      CHI=SQRT(1.-COCHI*COCHI)
      XPTB=RTBM*CHI
      THTB=ACOS((Q1*XTBS+Q2*YTBS+Q3*ZTBS)/(ABQ1*RTBS))
      DXTB=XPTB*COS(THTB)
      DXTB=-DXTB
      DYTB=XPTB*SIN(THTB)-RTBS
      DZTB=HRM/ABH1
C     COMPUTE DEVIATIONS FROM TWO-BODY RELATIVE MOTION DEVX,DEVY,DEVZ
      DEVX=DELTAX-DXTB
      DEVY=DELTAY-DYTB
      DEVZ=DELTAZ-DZTB
      PLOT1 (IPT) = DELTAX
      PLOT2 (IPT) = DELTAY
      PLOT3 (IPT) = DELTAZ
      PLOTZ(IPT)=DELTAP
C     NORMALIZE DEVX,DEVY,DEVZ BY 10**3---THEY ARE NOW IN METERS
      PLOT5(IPT)=DEVX *1.D+3
      PLOT6(IPT)=DEVY*1.D+3
      PLOT7(IPT)=DEVZ*1.D+3
      PLOT8(IPT)=AIPS
      PLOT9(IPT)=EIPS
      PLOT10(IPT)=FINCPS
      PLOT11(IPT)=CAPWS
      PLOT12(IPT)=SMAWS
      PLOT13(IPT)=PNUIS
      PLOT14(IPT)=MEANPS
      PLOT15(IPT)=AIPM
      PLOT16(IPT)=EIPM
      PLOT17(IPT)=FINCPM
      PLOT18(IPT)=CAPWM
      PLOT19(IPT)=SMAWM
```

```
      PLOT20(IPT)=PNUIM
      PLOT21(IPT)=MEANPM
C     PRINT ORBITAL ELEMENTS,TIME,AND RELATIVE MOTION COORDINATES
      WRITE(6,60) PNUIS, AIPS, EIPS, FINCPS, CAPWS, SMAWS, MEANPS,
     $ TTIMEM, TTIMED
   60 FORMAT(1H0, 8HPNUIS = ,E15.8 /
     $ 1X, 8HAIPS   = ,E15.8, 13X, 8HEIPS   = ,E15.8, 6X, 8HFINCPS= ,
     $ E15.8, 5X, 8HCAPWS = ,E15.8/1X, 8HSMAWS = ,E15.8, 13X, 8HMEANPS=
     $ E15.8 / 9H TTIMEM= ,E15.8, 13X, 8HTTIMED= ,E15.8)
C
      WRITE(6,70) PNUIM, AIPM, EIPM, FINCPM, CAPWM, SMAWM, MEANPM
   70 FORMAT(1H0, 8HPNUIM = ,E15.8/
     $ 1X, 8HAIPM   = ,E15.8, 13X, 8HEIPM   = ,E15.8, 6X, 8HFINCPM= ,
     $ E15.8, 5X, 8HCAPMS = ,E15.8 /1X, 8HSMAWM = ,E15.8, 13X,
     $ 8HMEANPM= ,E15.8)
C
      WRITE(6,781) DELTAX, DELTAY, DELTAZ
  781 FORMAT(1H0, 8HDELTAX= ,E15.8, 13X, 8HDELTAY= ,E15.8, 6X, 8HDELTAZ=
     $ ,E15.8)
C
      WRITE(6,1000) DXTB,DYTB,DZTB
 1000 FORMAT(1X, 8HDEVX   = ,E15.8, 13X, 8HDEVY   = ,E15.8, 6X, 8HDEVZ   =
     $ E15.8///)
      XNDTP=NDTP
      TTIME=TTIME+XNDTP*DT
      XY=TTIME/3600.
      IF(XY-TCUT) 101,101,102
  102 CONTINUE
C     PLOT PERTURBED ORBITAL ELEMENTS AND RELATIVE POSITION COORDINATES
      WRITE(6,2000)
 2000 FORMAT(1H0/1H0, 18HENTERING PLOT AREA)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB1 , -IPT, PLOT, PLOT1 )
      CALL QUIK3V (-1, 1H*, XLAB, YLAB2 , -IPT, PLOT, PLOT2 )
      CALL QUIK3V (-1, 1H*, XLAB, YLAB3 , -IPT, PLOT, PLOT3 )
      CALL QUIK3V(-1, 1H*, XLAB, YLAB4 , -IPT, PLOT, PLOTZ )
      K=0
      VALUE=PLOT1(1)
      DO 1 KP=1,IPT
      IF(PLOT1(KP)-VALUE) 601,600,600
  600 K=K+1
```

```
      PLOTX(K)=PLOT1(KP)
      PLOTY(K)=PLOT2(KP)
  1   CONTINUE
  601 CALL QUIK3V(-1, 1H*, YLAB1, YLAB2, - K , PLOTX, PLOTY)
      CALL QUIK3V(-1, 1H*, XLAB, YLAB5,-IPT, PLOT, PLOT5)
      CALL QUIK3V(-1, 1H*, XLAB, YLAB6,-IPT, PLOT, PLOT6)
      CALL QUIK3V(-1, 1H*, XLAB, YLAB7,-IPT, PLOT, PLOT7)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB8, -IPT, PLOT, PLOT8)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB9, -IPT, PLOT, PLOT9)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB10,-IPT, PLOT, PLOT10)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB11,-IPT, PLOT, PLOT11)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB12,-IPT, PLOT, PLOT12)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB13,-IPT, PLOT, PLOT13)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB14,-IPT, PLOT, PLOT14)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB15,-IPT, PLOT, PLOT15)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB16,-IPT, PLOT, PLOT16)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB17,-IPT, PLOT, PLOT17)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB18,-IPT, PLOT, PLOT18)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB19,-IPT, PLOT, PLOT19)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB20,-IPT, PLOT, PLOT20)
      CALL QUIK3V (-1, 1H*, XLAB, YLAB21,-IPT, PLOT, PLOT21)
      WRITE(6,2001)
 2001 FORMAT(1H0, 20HPLOTTING IS FINISHED)
      CALL ENDJOB
      STOP
      END
'FOR,IS  COWELL,COWELL
      SUBROUTINE COWELL(A,CDA,FINCP,CAPWP,SMAWP,MEANP,AIP,EIP,PNUI,Q,AX,
     *T)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/XK/NP,NDTP,H
      DIMENSION A(14),AX(14),B(15)
      DOUBLE PRECISION MEANP,MEAN
C     THIS ROUTINE INTEGRATES THE PERTURBED EQUATIONS OF MOTION USING
C     A FOURTH ORDER RUNGE-KUTTA SCHEME
      N=3
 1    M=0
C     CALL MODSTA TO GET INITIAL VALUES OF ACCELERATION
      CALL MODSTA(1,T,A,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,R,CDA)
C     CALL MODSTA TO COMPUTE ORBITAL ELEMENTS FOR INITIAL AND SUBSE-
```

```
C     QUENT TIME PERIODS .
      CALL MODSTA(2,T,A,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,R,CDA)
C     STORE POSITION,VELOCITY,ACCELERATION AND ORBITAL ELEMENTS TO BE
C     RETURNED TO MAIN ROUTINE
      Q=R
      AIP = AXIS
      EIP = ECCEN
      PNUI = ANOM
      CAPWP=CAPW
      SMAWP=SMAW
      FINCP = FINC
      MEANP = MEAN
      AX(1)=A(1)
      AX(4)=A(4)
      AX(7)=A(7)
      AX(2)=A(2)
      AX(5)=A(5)
      AX(8)=A(8)
C     COMPUTE FIRST HALF-STEP THE FIRST TIME
      C=H/2.
      K=3*N
      D = C**2 / 2.0
      E = D * 4.0
      F = C / 3.0
      T=T+C
      L=1
      DO 2 I=1,K,3
      B(L)=A(I)
      B(L+1)=A(I+1)
      B(L+2)=A(I+2)
      A(I)=A(I)+C*A(I+1)+D*A(I+2)
      A(I+1)=A(I+1)+C*A(I+2)
    2 L=L+6
C     COMPUTE FIRST HALF-STEP SECOND TIME
      CALL MODSTA(1,T,A,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,R,CDA)
      L=1
      DO 3 I=1,K,3
      B(L+3)=A(I+2)
      A(I+1)=B(L+1)+C*A(I+2)
    3 L=L+6
```



2-2

```
C       COMPUTE SFCOND HALF-STFP THF FIRST TIME
        CALL MODSTA(1,T,A,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,R,CDA)
        L=1
        T=T+C
        DO 4 I=1,K,3
        P(L+4)=A(I+2)
        A(I+1)=H*A(I+2)+B(L+1)
        A(I)=B(L)+H*B(L+1)+E*A(I+2)
   4    L=L+5
C       COMPUTE FINAL CONDITIONS FOR THE TIME STEP
        CALL MODSTA(1,T,A,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,R,CDA)
        L=1
        DO 5 I=1,K,3
        A(I)=B(L)+H*(B(L+1)+F*(B(L+2)+B(L+3)+B(L+4)))
        A(I+1)=B(L+1)+F*(B(L+2)+A(I+2)+2.*(B(L+3)+B(L+4)))
   5    L=L+5
        IF(NP-NDTP)1,37,37
  37    RETURN
        END
'FOR,IS  MODSTA,MODSTA
        SUBROUTINE MODSTA(JJ,T,A,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,R,CDA
       *)
        IMPLICIT REAL*8 (A-H,O-Z)
        COMMON/XK/NP,NDTP,DT
C       THIS ROUTINE DETERMINES THE TOTAL ACCELERATION COMPONENTS AND
C       ALSO COMPUTES ORBITAL ELEMENTS FROM POSITION AND VELOCITY
        LOGICAL SETI
        DOUBLE PRECISION MEAN
        DIMENSION A(14)
        SETI=.FALSE.
        PIE=3.141591
        DPR=57.2957795
        A2=6378.165
        W=6356.784
        IF(JJ-1) 702,702,905
  702   R= SQRT(A(1)**2+A(4)**2+A(7)**2)
C****   COMPUTE ALTITUDE OF VEHICLE ABOVE EARTH,S SURFACE
        AK=(W**2)/(A2**2)
        BK=AK**2
  55    RHA= SQRT(A(1)**2+A(4)**2)
```

```
      DEL= ATAN(A(7)/RHA)
      PSI= ATAN(AK* SIN(DEL)/ COS(DEL))
      SBI=( SIN(PSI)/ COS(PSI))**2
      SCI=SQRT((1.+BK*SBI)/(1.+AK*SBI))
      RI=A2*SCI
      ALT=R-RI
      IF(ALT-1000.) 65,71,71
   71 RHO=0.
      GO TO 66
   65 CONTINUE
C**** CALL SETUP TO DETERMINE DENSITY AT GIVEN ALTITUDE
      CALL SETUP(A,ALT,T,RHO)
   66 OME=7.29211585494E-05
      VE= SQRT((A(2)+OME*A(4))**2+(A(5)-OME*A(1))**2+A(8)**2)
      FDRG=-.5*CDA*RHO*VE*1.E+3
C**** COMPUTE DRAG ACCELERATION COMPONENTS
      XDDDR=FDRG*(A(2)+OME*A(4))
      YDDDR=FDRG*(A(5)-OME*A(1))
      ZDDDR=FDRG*A(8)
  936 R2=R*R
      R3=R2*R
      R5=R3*R2
      R7=R5*R2
      B=A(7)*A(7)/R2
      BBBB=9.0   *B*B
C**** COMPUTE TWO-BODY ACCELERATION COMPONENTS
      X2BODY=-3.986032E+05*A(1)/R3
      Y2BODY=-3.986032E+05*A(4)/R3
      Z2BODY=-3.986032E+05*A(7)/R3
C**** COMPUTE EFFECTS OF EARTH,S OBLATENESS(J2,J3,J4)
      XJ20B=-2.63251708557E+10*A(1)*(1.-5.*B)/R5
      XJ30B=+5.94697175476E+11*A(1)*(3.-7.*B)*A(7)/R7
      XJ40B=-5.19486592772E+15*A(1)*(.42857142857-6.*B+BBBB)/R7
      YJ20B=-2.63251708557E+10*A(4)*(1.-5.*B)/R5
      YJ30B=+5.94697175476E+11*A(4)*A(7)*(3.-7.*B)/R7
      YJ40B=-5.19486592772E+15*A(4)*(.42857142857-6.*B+BBBB)/R7
      ZJ20B=-2.63251708557E+10*A(7)*(3.-5.*B)/R5
      ZJ30B=-3.568183053E+11*(1.-10.*B+11.6666666667*B*B)/R5
      ZJ40B=-5.19486592772E+15*A(7)*(2.1428571428-10.*B+BBBB)/R7
C**** COMPUTE TOTAL ACCELERATION COMPONENTS
```

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```
      A(3)=X2BODY+XJ20B+XJ30B+XJ40B+XDDDR
      A(6)=Y2BODY+YJ20B+YJ30B+YJ40B+YDDDR
      A(9)=Z2BODY+ZJ20B+ZJ30B+ZJ40B+ZDDDR
  902 RETURN
  905 NP=NP+1
      IF (NDTP-NP) 904,904,903
  904 CONTINUE
C**** COMPUTE OSCULATING ORBITAL ELEMENTS.
      XKERTH = 398603.2
      HX=A(4)*A(8)-A(7)*A(5)
      HY=A(7)*A(2)-A(1)*A(8)
      HZ=A(1)*A(5)-A(4)*A(2)
      HSQ=SQRT(HX*HX+HY*HY+HZ*HZ)
      C3=A(2)*A(2)+A(5)*A(5)+A(8)*A(8)-2.*XKERTH/R
C**** SEMI-MAJOR AXIS
      AXIS=-XKERTH/C3
      XN=SQRT(XKERTH/AXIS**3)
C**** ECCENTRICITY
      ECCEN=1.+HSQ*HSQ*C3/XKERTH/XKERTH
      IF(ECCEN)10,30,20
   10 ECCEN=0.
      GO TO 30
   20 ECCEN=SQRT(ECCEN)
      IF(ECCEN-.00001)10,10,30
C**** INCLINATION
   30 FINC=ATAN2(SQRT(HX*HX+HY*HY),HZ)
      IF(FINC*DPR-.00001)40,40,50
   40 FINC=0.
      SETI=.TRUE.
      GO TO 70
   50 IF(FINC*DPR-179.99999) 80,60,60
   60 FINC=PIE
      SETI=.TRUE.
   70 CAPW=0.
      GO TO 100
C**** ASCENDING NODE
   80 CAPW=ATAN2(HX,(-HY))
  100 TEMP=HSQ*HSQ/XKERTH
C**** TRUE ANOMALY
      ANOM=R*R*ECCEN*ECCEN-(TEMP-R)*(TEMP-R)
```

```
      IF(ANOM.GT.0.) GO TO 120
  110 ANOM=0.
      GO TO 140
  120 IF(ECCEN) 101,110,101
  101 CONTINUE
      ANOM=ATAN2(SQRT(ANOM),TEMP-R)
      TEMP=(A(1)*A(2)+A(4)*A(5)+A(7)*A(8))/R
      IF(TEMP.LT.0.) GO TO 130
      ANOM=ABS(ANOM)
      GO TO 140
  130 ANOM=-ABS(ANOM)
  140 TEMP2=A(1)*COS(CAPW)+A(4)*SIN(CAPW)
      TEMP=R*R-TEMP2*TEMP2
      IF(TEMP.GT.0.)GO TO 150
      TEMP3=0.
      GO TO 190
  150 TEMP3=ATAN2(SQRT(TEMP),TEMP2)
      IF(A(7).LT.0.) GO TO 170
  160 TEMP3=ABS(TEMP3)
      GO TO 190
  170 IF(ECCEN)171,180,171
  171 CONTINUE
      IF(A(7)) 180,160,180
  180 TEMP3=-ABS(TEMP3)
  190 IF(ECCEN) 250,200,250
  200 IF(SETI) GO TO 240
      IF(A(7)) 209,210,209
C**** ARGUMENT OF PERIGEE
  209 CONTINUE
      SMAW=TEMP3
      GO TO 260
  210 IF(A(8)) 220,220,230
  220 SMAW=0.
      GO TO 260
  230 SMAW=PIE
      GO TO 260
  240 SMAW=ATAN2(A(4),A(1))
      GO TO 260
  250 SMAW=TEMP3-ANOM
  260 CONTINUE
```

```
C**** MEAN ANOMALY
      MEAN=XN*T
      MEAN=AMOD(MEAN,6.2831854)
      MEAN=MEAN*DPR
      ANOM=AMOD(ANOM,6.2831854)
      SMAW=AMOD(SMAW,6.2831854)
      CAPW=AMOD(CAPW,6.2831854)
12012 CONTINUE
      ANOM = ANOM * DPR
  903 RETURN
      END
'FOR,IS  TRANFM,TRANFM
      SUBROUTINE TRANFM(AXIS,ECCEN,INC,ASNOD,ANOM,ARGP,XS,YS,ZS,XDS,YDS,
     $ZDS)
      IMPLICIT REAL*8 (A-H,O-Z)
      DOUBLE PRECISION INC
C**** THIS ROUTINE TRANSFORMS OSCULATING ORBITAL ELEMENTS TO POSITION
C     AND VELOCITY COMPONENTS IN A SPACE FIXED EPHEMERIS COORDINATE
C     SYSTEM(EARTH-CENTERED
      XKERTH=398603.2
      DPR=57.2957795
      INC=INC/DPR
      ASNOD=ASNOD/DPR
      ANOM=ANOM/DPR
      ARGP=ARGP/DPR
      TEMP1=(AXIS*(1.-ECCEN*ECCEN))/(1.+ECCEN*COS(ANOM))
      TEMP2=TEMP1*SIN(ARGP+ANOM)
      TEMP3=TEMP1*COS(ARGP+ANOM)
      TEMP4=TEMP2*COS(INC)
      TEMP5=TEMP2*SIN(INC)
      TEMP6=SIN(ASNOD)
      TEMP7=COS(ASNOD)
      XS=TEMP3*TEMP7-TEMP4*TEMP6
      YS=TEMP3*TEMP6+TEMP4*TEMP7
      ZS=TEMP5
      TEMP=2.*XKERTH/TEMP1-XKERTH/AXIS
      IF(TEMP.GT.0.)GO TO 20
      TEMP10=0.
      GO TO 30
   20 TEMP10=SQRT(TEMP)
```

```
   30 TEMP11=ATAN2(ECCEN*SIN(ANOM),1.+ECCEN*COS(ANOM))
      TEMP8=SIN(ARGP+ANOM)
      TEMP9=COS(ARGP+ANOM)
      TEMP2=TEMP10*(SIN(TEMP11)*TEMP8+COS(TEMP11)*TEMP9)
      TEMP3=TEMP10*(SIN(TEMP11)*TEMP9-COS(TEMP11)*TEMP8)
      TEMP4=TEMP2*COS(INC)
      TEMP5=TEMP2*SIN(INC)
      XDS=TEMP3*TEMP7-TEMP4*TEMP6
      YDS=TEMP3*TEMP6+TEMP4*TEMP7
      ZDS=TEMP5
      RETURN
      END
'FOR,IS  BLOK,BLOK
      BLOCK DATA
      COMMON/DATAB/XLAB(12),YLAB1(12),YLAB2(12),YLAB3(12),YLAB4(12),
     1YLAB5(12),YLAB6(12),YLAB7(12),YLAB8(12),YLAB9(12),YLAB10(12),
     2YLAB11(12),YLAB12(12),YLAB13(12),YLAB14(12),YLAB15(12),YLAB16(12),
     3YLAB17(12),YLAB18(12),YLAB19(12),YLAB20(12),YLAB21(12)
      DATA XLAB   /6H      , 6H      , 6H TIME , 6HIN MIN, 6HUTES  ,
     $ 6H      ,  6H      , 6H      , 6H      , 6H      , 6H      ,
     $ 6H      /
C
      DATA YLAB1 /6H      , 6H   X  , 6H IN KM, 6HS.    , 6H      ,
     $ 6H      ,  6H      , 6H      , 6H      , 6H      , 6H      ,
     $ 6H      /
C
      DATA YLAB2 /6H      , 6H   Y  , 6H IN KM, 6HS.    , 6H      ,
     $ 6H      ,  6H      , 6H      , 6H      , 6H      , 6H      ,
     $ 6H      /
C
      DATA YLAB3 /6H      , 6H   Z  , 6H IN KM, 6HS.    , 6H      ,
     $ 6H      ,  6H      , 6H      , 6H      , 6H      , 6H      ,
     $ 6H      /
      DATA YLAB4 /6H      , 6H   R  , 6H IN KM, 6HS.    , 6H      ,
     $ 6H      ,  6H      , 6H      , 6H      , 6H      , 6H      ,
     $ 6H      /
      DATA YLAB5 /6H      , 6HX  MIN,6HUS TWO,6HBODY X,6H      ,
     $6H      ,6H      ,6H      ,6H      ,6H      ,6H      ,6H      /
      DATA YLAB6 /6H      , 6HY   MI, 6HNUS   , 6HY TWOB, 6HODY   ,
     $ 6H      , 6H      , 6H      , 6H      , 6H      , 6H      ,
```

```
$ 6H      /
 DATA YLAB7 /6H        , 6HZ  MIN,6HUS TWO,6HBODY Z,6H      ,
$6H      ,6H      ,6H      ,6H      ,6H      ,6H      ,6H
 DATA YLAB8 /6H        , 6HSTATIO, 6HN AXIS, 6H IN KM, 6HS.    ,
$ 6H      , 6H      , 6H      , 6H      , 6H      , 6H      ,
$6H      /
 DATA YLAB9 /6H        , 6HSTATIO, 6HN ECCE, 6HNTRICI, 6HTY    ,
$ 6H      , 6H      , 6H      , 6H      , 6H      , 6H      ,
$ 6H      /
 DATA YLAB10/6H        , 6HSTATIO, 6HN INCL, 6HINATIO, 6HN IN D,
$ 6HEGS.  , 6H      , 6H      , 6H      , 6H      , 6H      ,
$ 6H      /
 DATA YLAB11/6H        , 6HSTATIO, 6HN NODE, 6H IN DE, 6HGS.   ,
$6H      ,6H      ,6H      ,6H      ,6H      ,6H      ,
$ 6H      /
 DATA YLAB12/6H        , 6HSTATIO, 6HN ARG ,6H OF PE, 6HRIGEE ,
$ 6HIN DEG, 6HS.    , 6H      , 6H      , 6H      , 6H      ,
$ 6H      /
 DATA YLAB13/6H        , 6HSTATIO, 6HN TRUE, 6H ANOMA, 6HLY IN ,
, 6HDEGS. , 6H      , 6H      , 6H      , 6H      , 6H      ,
$ 6H      /
 DATA YLAB14/6H        , 6HSTATIO, 6HN MEAN, 6H ANOMA, 6HLY IN ,
$ 6HDEGS. , 6H      , 6H      , 6H      , 6H      , 6H      ,
$ 6H      /
 DATA YLAB15/6H        ,6HMODULE, 6H AXIS ,6HIN KMS, 6H.    ,
$ 6H      , 6H      , 6H      , 6H      , 6H      , 6H      ,
$ 6H      /
 DATA YLAB16/6H        ,6HMODULE, 6H ECCEN, 6HTRICIT,6HY     ,
$ 6H      , 6H      , 6H      , 6H      , 6H      , 6H      ,
$ 6H      /
 DATA YLAB17/6H        ,6HMODULE, 6H INCLI, 6HNATION,6H IN DE,
$ 6HGS.   , 6H      , 6H      , 6H      , 6H      ,6H      ,
$6H      /
 DATA YLAB18/6H        ,6HMODULE, 6H NODE ,6HIN DEG, 6HS.    ,
$ 6H      , 6H      , 6H      , 6H      , 6H      , 6H      ,
$ 6H      /
 DATA YLAB19/6H        ,6HMODULE, 6H ARG O,6HF PERI, 6HGEE IN,
$ 6HDEGS. , 6H      , 6H      , 6H      , 6H      ,6H      ,
$ 6H      /
 DATA YLAB20/6H        ,6HMODULE, 6H TRUE ,6HANOMAL, 6HY IN D,
```

```
     • 6HEGS•   • 6H       • 6H       • 6H       • 6H       • 6H       •
     • 6H       /
      DATA YLAB21/6H      •6HMODULE• 6H MEAN •6HANOMAL• 6HY IN D•
     $6HEGS•   • 6H       • 6H       • 6H       • 6H       • 6H       •
     $6H       /
      END
'FOR,IS  TOBODY,TOBODY
      SUBROUTINE TOBODY(A,AX,Q,AXIS,S,XI,CAP,SMAW,TRUE,MEANP,T)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(14),AX(14),B(15)
      DOUBLE PRECISION MEANP,MEAN
      COMMON/XK/NP,NDTP,H
      N=3
C     THIS ROUTINE INTEGRATES THE EQUATIONS OF MOTION USING A FOURTH
C     ORDER RUNGE-KUTTA NUMERICAL SCHEME
    1 CONTINUE
C     CALL CONIC  TO GET INITIAL VALUES OF ACCELERATION
      CALL CONIC(1,A,R,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,T)
C     CALL CONIC  TO COMPUTE ORBITAL ELEMENTS FOR INITIAL AND SUBSE-
C     QUENT TIME PERIODS
      CALL CONIC(2,A,R,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,T)
C     STORE POSITION,VELOCITY,ACCELERATION AND ORBITAL ELEMENTS TO BE
C     RETURNED TO MAIN ROUTINE
      Q=R
      AX(1)=A(1)
      AX(2)=A(2)
      AX(4)=A(4)
      AX(5)=A(5)
      AX(7)=A(7)
      AX(8)=A(8)
      AXIS=AXIS
      S=ECCEN
      XI=FINC
      CAP=CAPW
      SMAW=SMAW
      MEANP=MEAN
      TRUE=ANOM
C     COMPUTE FIRST HALF-STEP THE FIRST TIME
      C=H/2.
      K=3*N
```

```
      D=C**2/2.
      E=D*4.
      F=C/3.
      T=T+C
      L=1
      DO 2  I=1,K,3
      B(L)=A(I)
      B(L+1)=A(I+1)
      B(L+2)=A(I+2)
      A(I)=A(I)+C*A(I+1)+D*A(I+2)
      A(I+1)=A(I+1)+C*A(I+2)
    2 L=L+5
C     COMPUTE FIRST HALF-STEP SECOND TIME
      CALL CONIC(1,A,R,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,T)
      L=1
      DO 3  I=1,K,3
      B(L+3)=A(I+2)
      A(I+1)=B(L+1)+C*A(I+2)
    3 L=L+5
C     COMPUTE SECOND HALF-STEP THE FIRST TIME
      CALL CONIC(1,A,R,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,T)
      L=1
      T=T+C
      DO 4  I=1,K,3
      B(L+4)=A(I+2)
      A(I+1)=H*A(I+2)+B(L+1)
      A(I)=B(L)+H*B(L+1)+E*A(I+2)
    4 L=L+5
C     COMPUTE FINAL CONDITIONS FOR THE TIME STEP
      CALL CONIC(1,A,R,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,T)
      L=1
      DO 5  I=1,K,3
      A(I)=B(L)+H*(B(L+1)+F*(B(L+2)+B(L+3)+B(L+4)))
      A(I+1)=B(L+1)+F*(B(L+2)+A(I+2)+2.*(B(L+3)+B(L+4)))
    5 L=L+5
      IF(NP-NDTP)1,37,37
   37 RETURN
      END
'FOR,IS  CONIC,CONIC
      SUBROUTINE CONIC(JJ,A,R,AXIS,ECCEN,ANOM,CAPW,SMAW,MEAN,FINC,T)
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/XK/NP,NDTP,DT
      DOUBLE PRECISION MEAN
      LOGICAL SFTI
C     THIS ROUTINE DETERMINES THE TOTAL ACCELERATION COMPONENTS AND
C     ALSO COMPUTES ORBITAL ELEMENTS FROM POSITION AND VELOCITY
      DIMENSION A(14)
      SFTI=.FALSE.
      PIE=3.141591
      DPR=57.2957795
      IF(JJ-1) 702,702,905
  702 R= SQRT(A(1)**2+A(4)**2+A(7)**2)
  936 R2=R*R
      R3=R2*R
C**** COMPUTE TWO-BODY ACCELERATION COMPONENTS
      A(3)=-3.986032E+05*A(1)/R3
      A(6)=-3.986032E+05*A(4)/R3
      A(9)=-3.986032E+05*A(7)/R3
  902 RETURN
  905 NP=NP+1
      IF (NDTP-NP) 904,904,903
  904 CONTINUE
C**** COMPUTE OSCULATING ORBITAL ELEMENTS
      XKERTH = 398603.2
      HX=A(4)*A(8)-A(7)*A(5)
      HY=A(7)*A(2)-A(1)*A(8)
      HZ=A(1)*A(5)-A(4)*A(2)
      HSQ=SQRT(HX*HX+HY*HY+HZ*HZ)
      C3=A(2)*A(2)+A(5)*A(5)+A(8)*A(8)-2.*XKERTH/R
C**** SEMI-MAJOR AXIS
      AXIS=-XKERTH/C3
      XN=SQRT(XKERTH/AXIS**3)
C**** ECCENTRICITY
      ECCEN=1.+HSQ*HSQ*C3/XKERTH/XKERTH
      IF(ECCEN)10,30,20
   10 ECCEN=0.
      GO TO 30
   20 ECCEN=SQRT(ECCEN)
      IF(ECCEN-.00001)10,10,30
C**** INCLINATION
```

```
   30 FINC=ATAN2(SQRT(HX*HX+HY*HY),HZ)
      IF(FINC*DPR-.00001)40,40,50
   40 FINC=0.
      SETI=.TRUE.
      GO TO 70
  50  IF(FINC*DPR-179.99999) 80,60,60
   60 FINC=PIE
      SETI=.TRUE.
   70 CAPW=0.
      GO TO 100
C**** ASCENDING NODE
   80 CAPW=ATAN2(HX,(-HY))
  100 TEMP=HSQ*HSQ/XKEPTH
C**** TRUE ANOMALY
      ANOM=R*R*ECCEN*ECCEN-(TEMP-R)*(TEMP-R)
      IF(ANOM.GT.0.) GO TO 120
  110 ANOM=0.
      GO TO 140
  120 IF(ECCEN) 101,110,101
  101 CONTINUE
      ANOM=ATAN2(SQRT(ANOM),TEMP-R)
      TEMP=(A(1)*A(2)+A(4)*A(5)+A(7)*A(8))/R
      IF(TEMP.LT.0.) GO TO 130
      ANOM=ABS(ANOM)
      GO TO 140
  130 ANOM=-ABS(ANOM)
  140 TEMP2=A(1)*COS(CAPW)+A(4)*SIN(CAPW)
      TEMP=R*R-TEMP2*TEMP2
      IF(TEMP.GT.0.)GO TO 150
      TEMP3=0.
      GO TO 190
  150 TEMP3=ATAN2(SQRT(TEMP),TEMP2)
      IF(A(7).LT.0.) GO TO 170
  160 TEMP3=ABS(TEMP3)
      GO TO 190
  170 IF(ECCEN)171,180,171
  171 CONTINUE
      IF(A(7)) 180,160,180
  180 TEMP3=-ABS(TEMP3)
  190 IF(ECCEN) 250,200,250
```

```
200 IF(SETI) GO TO 240
    IF(A(7)) 209,210,209
*** ARGUMENT OF PERIGEE
209 CONTINUE
    SMAW=TEMP3
    GO TO 260
210 IF(A(8)) 220,220,230
220 SMAW=0.
    GO TO 260
230 SMAW=PIE
    GO TO 260
240 SMAW=ATAN2(A(4),A(1))
    GO TO 260
250 SMAW=TEMP3-ANOM
260 CONTINUE
*** MEAN ANOMALY
    MEAN=XN*T
    MEAN=AMOD(MEAN,6.2831854)
    MEAN=MEAN*DPR
    ANOM=AMOD(ANOM,6.2831854)
    SMAW=AMOD(SMAW,6.2831854)
    CAPW=AMOD(CAPW,6.2831854)
'012 CONTINUE
    ANOM = ANOM * DPR
903 RETURN
    END
'OR,IS  SETUP,SETUP
    SUBROUTINE SETUP(A,ALT,TIME,RHO)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION A(14),UBAR(3),FTENB(200)
    THIS ROUTINE SETS UP ALL INFORMATION NECESSARY TO COMPUTE DENSITY
    IN ROUTINE JACHIA
    DATA DPR/57.29578D+0/,ECLIPT/23.4436D+0/,FLAG/0.0D+0/,ENDID/6HEND
   $ /
    IF(FLAG)8,10,8
10  CONTINUE
    READ MODIFIED JULIAN DATE = JULIAN DATE - 2400000.5
    READ(5,105)XJD
105 FORMAT(D12.8)
    READ THE NUMBER OF INPUT DATA TO BE FOUND ON SUCCEEDING CARDS
```

```
    READ(5,100)K
100 FORMAT(3I3)
    DO 76 I=1,K,6
    READ DECIMETRIC SOLAR FLUX(AVE. VALUE),DECIMAL YEAR,FLUX,YEAR,ETC
 76 READ(5,101) FTENB(I),FTENB(I+1),FTENB(I+2),FTENB(I+3),FTENB(I+4),
   *FTENB(I+5)
101 FORMAT(6D12.8)
    FTENB(K+1)=ENDID
    XDAYS=XJD-36203.
8   CONTINUE
    XDAYS=XDAYS+TIME/86400.
    DAYNO=AMOD(XDAYS,365.25)
    YERR=1958.+(XDAYS/365.25)
    DETERMINEVALUE OF SOLAR FLUX FOR DATE IN QUESTION
    M=1
030 IF(FTENB(M)-ENDID)1050,1040,1050
040 F10B=FTENB(M-2)
    GO TO 1100
050 IF(FTENB(M+1)-YERR)1060,1070,1080
060 M=M+2
    GO TO 1030
070 F10B=FTENB(M)
    GO TO 1100
080 IF(M-1)1070,1070,1090
090 F10B=FTENB(M-2)+(YERR-FTENB(M-1))*(FTENB(M)-FTENB(M-2))/(FTENB(M+1
   *)-FTENB(M-1))
100 CONTINUE
    DETERMINE GEOMAGNETIC INDEX,AP,FROM SOLAR FLUX NUMBER
260 IF(F10B.GE.130.) GO TO 1280
    IF(F10B.GE.80.) GO TO 1270
    AP=6.
    GO TO 1440
270 AP=8.
    GO TO 1440
280 AP=11.
440 CONTINUE
    F10=F10B
    XLAMS=.017203*XDAYS+.0335*SIN(.017203*XDAYS)-1.41
001 IF(XLAMS-6.2831853)10003,10003,10002
002 XLAMS=XLAMS-6.2831853
```

```
      GO TO 10001
)03   CONTINUE
      XLS=COS(XLAMS)
      TFMO=SIN(XLAMS)
      TFMR=COS(ECLIPT/DPR)
      TFMS=SIN(ECLIPT/DPR)
      XMS=TFMR*TFMO
      XNS=TFMS*TFMO
      COMPUTE RIGHT ASCENSION AND DECLINATION OF SUN
      ALSUN=ATAN2(XMS,XLS)
      DLSUN=ASIN(XNS)
      UBAR(1)=A(1)
      UBAR(2)=A(4)
      UBAR(3)=A(7)
      HKM=ALT
      CALL DENSITY ROUTINE
      CALL JACHIA(HKM,F10,F10B,ALSUN,DLSUN,AP,UBAR,DAYNO,RHO)
      RHO=1.D+3*RHO
      FLAG=1.
      RETURN
      END
R,IS   JACHIA,JACHIA
      SUBROUTINE JACHIA(HKM,F10,F10BAR,ALSUN,DLSUN,AP,UBAR,DAYNO,RHO)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON FN(5),Y(5),COE(6)
      DIMENSION UBAR(3),LOGRHO(4),SBUF(4)
      REAL LOGRHO,LGRHOH
      DIMENSION TABL(80),TABL1(40)
      THIS ROUTINE COMPUTES DENSITY(GM/CM**3) FROM THE MSFC MODIFIED
      JACCHIA MODEL ATMOSPHERE(1967) AS GIVEN IN --SPACE ENVIRONMENT
      CRITERIA GUIDELINES FOR USE IN SPACE VEHICLE DEVELOPMENT(1969
      REVISION),DON K. WEIDER,EDITOR
      DATA TABL /
     $-0.29118614D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
     $-0.33835106D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
     $-0.40510482D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
     $-0.47349510D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
     $-0.53983993D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
     $-0.59884720D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
     $-0.65143890D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
```

```
$-0.70578155D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
$-0.76952820D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
$ 44*0.0D+0/
 DATA TABL1 /
$-0.93030013D+01,+0.0        D+00,+0.0        D+00,+0.0        D+00,
$-0.10007000D+02,+0.0        D+00,+0.0        D+00,+0.0        D+00,
$-0.10609000D+02,-0.47683716D-06,-0.47683716D-06,+0.0        D+00,
$-0.11091000D+02,-0.31907082D-01,+0.15483856D-01,-0.25072098D-02,
$-0.11411000D+02,-0.24660110D-01,+0.70843697D-02,-0.59747696D-03,
$-0.11656000D+02,-0.49915314D-02,-0.86374283D-02,+0.28266907D-02,
$-0.11857000D+02,-0.20017624D-01,-0.26494980D-01,+0.66037178D-02,
$-0.12032000D+02,-0.47489643D-01,-0.44704437D-01,+0.10418415D-01,
$-0.12187000D+02,-0.76425552D-01,-0.62486649D-01,+0.13999939D-01,
* 4*0.0D+0/
 EQUIVALENCE (TABL(41), TABL1(1))
 TABL(37)=-0.84989394E+01
 TABL(38)=-0.0
 TABL(39)=-0.0
 TABL(40)=-0.0
 TABL1(37)=-0.12329000E+02
 TABL1(38)=+0.10565329E-00
 TABL1(39)=-0.79087734E-01
 TABL1(40)=+0.17275810E-01
 COE(1)=-10.48947029
 COE(2)=2.844291123E-02
 COE(3)=-3.620959821E-05
 COE(4)=2.341193059E-08
 COE(5)=-7.577509214E-12
 COE(6)=9.753963073E-16
 FK=8.31432E+07
 CONL=.43429448
 PI=3.14159268
 TPI=6.28318536
 PI4=.7853982
 CON=57.29577951
 EPS=23.45/CON
 F10B=F10BAR
 TOB=362.+3.6*F10B
 TOP=TOB+1.8*(F10-F10B)
 TO=TOP+(.37+.14*SIN(TPI*(DAYNO-151.)/365.))*
```

```
     1(FLOB*SIN(2.*TPI*(DAYNO-59.)/365.))
      RMAG=SQRT(UBAR(1)*UBAR(1)+UBAR(2)*UBAR(2)+UBAR(3)*UBAR(3))
      PHI=ASIN(UBAR(3)/RMAG)
      HSUN=ATAN2(UBAR(2),UBAR(1))
   10 IF(HSUN.GE.0.0) GO TO 20
      HSUN=HSUN+TPI
      GO TO 10
   20 IF(HSUN.LE.TPI) GO TO 30
      HSUN=HSUN-TPI
      GO TO 20
   30 HSUN=HSUN-ALSUN
31    IF(HSUN.GE.0.)GO TO 32
      HSUN=HSUN+TPI
      GO TO 31
   32 IF(HSUN.LT.TPI) GO TO 33
      HSUN=AMOD(HSUN,TPI)
   33 TAU=HSUN-PI4+.209439*SIN(HSUN+PI4)
      IF(TAU)270,250,250
  250 IF(TAU-PI)290,290,260
  260 TAU=TAU-TPI
      GO TO 250
  270 IF(TAU+PI)280,290,290
  280 TAU=TAU+TPI
      GO TO 270
  290    CONTINUE
  295 ETA=0.5*(PHI-DLSUN)
      PSI=ABS(0.5*(PHI+DLSUN))
  300 POW=2.5
  320 SPSI=SIN(PSI)
      CTAU=COS(TAU/2.0)
      CETA=COS(ETA)
      IF(SPSI)322,322,323
  322 SPSI=0.
      GO TO 325
  323 SPSI=SPSI**POW
  325 IF(CTAU)326,326,327
  326 CTAU=0.
      GO TO 330
  327 CTAU=CTAU**POW
  330 IF(CETA)331,331,332
```

```
  331 CETA=0.
      GO TO 333
  332 CETA=CETA**POW
  333 CONTINUE
      T=TO*(1.+.28*(SPSI+CTAU*CETA-CTAU*SPSI))
      DELT=AP+100.*(1.-EXP(-.08*AP))
      TF=T+DELT
      ERR=TF-800.
      X=ERR/(750.+1.722E-04*ERR**2)
      ST=.0291*EXP(-X**2/2.0)
      SIGMA=ST+.00015
  340 CONTINUE
      G=(HKM-120.)*6476.77/(6356.77+HKM)
      TZ=TE-(TE-355.)*EXP(-SIGMA*G)
      IF(HKM-120.)850,342,342
  850 TEMOD=(TE-1100.0)/400.0
      S=HKM/10.0
      I=S
      S=I
      S=(HKM/10.0)-S
      IF(S) 8501,110,8501
 8501 CONTINUE
      IF(I.LE.1) GO TO 70
      IF(I.GE.99) GO TO 80
      I=I*4-3
   50 J=I+12
      L=1
C     COMPUTE LOGRHO FOR 4 ALTITUDES
      DO 60 K=I,J,4
      LOGRHO(L)=TABL(K)+TABL(K+1)*TEMOD+TABL(K+2)*TEMOD*TEMOD+TABL(K+3)
     1*TEMOD*TEMOD*TEMOD
      L=L+1
   60 CONTINUE
      SBUF(1)=S*(S-1.0)*(S-2.0)
      SBUF(2)=(S-1.0)*(S-2.0)*(S+1.0)
      SBUF(3)=S*(S-2.0)*(S+1.0)
      SBUF(4)=S*(S-1.0)*(S+1.0)
C     COMPUTE LOG RHO AT H KM - INTERPOLATE
      LGRHOH=((SBUF(1)*LOGRHO(1))/(-6.0)) +
     1       ((SBUF(2)*LOGRHO(2))/2.0)    +
```

```
     2        ((SBUF(3)*LOGRHO(3))/(-2.0)) +
     3        ((SBUF(4)*LOGRHO(4))/6.0)
C      RHO=DENSITY(G/CM3) = ANTILOG OF LGRHOH
 65    RHO=10.**LGRHOH
       RETURN
   70  IF(I.LT.0) GO TO 90
       I=1
       GO TO 50
   80  IF(I.GT.100) GO TO 90
       I=389
       GO TO 50
   90  WRITE(6,100)HKM
  100  FORMAT(1H0,5H H = ,E15.8,5X,40HEXCEEDS ATMOSPHERE TABLE OF 0 TO 10
     *00 KM)
       RHO=0.
       RETURN
  110  J=I*4+1
       LGRHOH=TABL(J)+TABL(J+1)*TEMOD+TABL(J+2)*TEMOD*TEMOD+TABL(J+3)
      1*TEMOD*TEMOD*TEMOD
       GO TO 65
  342  FN(1)=4.0E+11
       FN(2)=7.5E+10
       FN(3)=7.6E+10
       FN(4)=3.4E+07
       IF(HKM-500.)350,345,345
  345  ERR=DLOG(TE)*CONL
       FN(5)=73.13-39.4*ERR+5.5*ERR**2
       FN(5)=10.0**FN(5)
       ALH=COE(1)+TE*(COE(2)+TE*(COE(3)+TE*(COE(4)+
      1TE*(COE(5)+TE*COE(6)))))
       N=5
       GO TO 360
  350  FN(5)=0.
       N=4
       IF(ABS(HKM-120.)-1.0E-08)380,380,360
  360  Y(5)=944.655E+05/(SIGMA*FK*TE)
       Y(1)=28.*Y(5)
       Y(2)=32.*Y(5)
       Y(3)=16.*Y(5)
       Y(4)=4.*Y(5)
```

```
      A=(TE-355.)/TE
      C=(1.-A)/(1.-A*EXP(-SIGMA*G))
      FN(1)=FN(1)*C**(1.+Y(1))*EXP(-SIGMA*Y(1)*G)
      FN(2)=FN(2)*C**(1.+Y(2))*EXP(-SIGMA*Y(2)*G)
      FN(3)=FN(3)*C**(1.+Y(3))*EXP(-SIGMA*Y(3)*G)
      FN(4)=FN(4)*C**(.63+Y(4))*EXP(-SIGMA*Y(4)*G)
      IF(ABS(HKM-500.)-1.0E-08)380,380,370
  370 FN(5)=FN(5)*C**(1.+ALH+Y(5))*EXP(-SIGMA*Y(5)*G)
  380 SUM=0.
      DO 390 I=1,N
      SUM=SUM+FN(I)
  390 CONTINUE
      ERR=28.*FN(1)+32.*FN(2)+16.*FN(3)+4.*FN(4)+FN(5)
      RHO=ERR*1.66E-24
      RETURN
      END
'MAP,IN   A,B
   LIB    SYS$*MSFC$.
'XQT P
7642.45       .1         55.0       0.0        0.0        0.0
7642.655777 0.1          55.0       0.0        0.0        0.0
 82       -02 205     -01
 3        +02 15      +02 10
 44359    +05
012
 13729    +03 1980    +04 13314   +03 198025  +04 12994   +03 19805   +04
 12690    +03 198075  +04 12371   +03 1981    +04 12112   +03 198125  +04
'FIN
'FIN
```